Empirical Properties of Diversion Ratios*

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Abstract

The diversion ratio for products j and k is the fraction of consumers who leave product j after a price increase and switch to product k. Theoretically, it is expressed as the ratio of demand derivatives from a multi-product firm's Bertrand-Nash first-order condition. In practice, diversion ratios are also measured from second-choice data or customer-switching surveys. We establish a LATE interpretation of diversion ratios, and show how diversion ratios are obtained from different interventions (price, quality, or assortment changes) and how those measures relate to one another and to underlying properties of demand.

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1. Introduction

The diversion ratio is one of the best ways economists have for understanding the nature of competition between sellers. As the price of j increases, some consumers leave product j, and a subset of these consumers switch to a substitute product k. The diversion ratio, D_{jk}, is defined as the ratio of the switchers to the leavers. Diversion ratios arise in the first-order conditions for Nash-in-prices games when sellers offer differentiated products. Two products with a high degree of differentiation face lower diversion and softer price competition, whereas two products with a high degree of similarity to competing goods face higher diversion and potentially tougher price competition.

Not surprisingly, diversion ratios are a central calculation of interest to antitrust authorities for analyzing horizontal mergers. The current U.S. merger guidelines, released in 2010, place greater weight on diversion ratios relative to concentration measures such as the Herfindahl-Hirschman Index (HHI).[1] In the context of merger reviews, antitrust authorities identify 'unilateral effects' as important for understanding the impact of a proposed merger. These arise when competition between the products of the merged firm is reduced due to the firm's post-merger ability to internalize substitution between its jointly-owned products.[2] This can lead to an increase in the price of the products of the merged firm, potentially harming consumers. Diversion ratios are the key statistic of interest for measuring unilateral effects. The current U.S. merger guidelines, released in 2010, note:

> *Diversion ratios between products sold by one merging firm and products sold by the other merging firm can be very informative for assessing unilateral price effects, with higher diversion ratios indicating a greater likelihood of such effects.*

[1] Researchers have pointed out concerns with using concentration measures or functions of market share to capture the strength of competition. For example, such measures require one to define a market and they do not capture the closeness of competition when products are differentiated.

[2] In contrast, the concept of harm via 'coordinated effects' arises if a proposed merger increases the incentives for firms in the industry to coordinate their behavior in an anti-competitive way.

Thus, holding competitive responses (and price-cost margins) fixed, antitrust agencies will be more concerned about mergers that involve products with higher diversion ratios, because the scope for price increases due to unilateral effects is thought to be greater.

Although the use of diversion ratios in antitrust policy is well understood theoretically, in practice, one needs to estimate diversion ratios. The U.S. merger guidelines discuss diversion ratios as calculated from an estimated demand system, or observed from consumer survey data or in a firm's course of business. We demonstrate that there is no single measure of diversion, just as there is no single measure of elasticity. Estimates of both diversion and elasticity depend on where one evaluates the demand curve. We provide a treatment-effects framework for interpreting different estimates of diversion. For a broad class of discrete-choice models (including both random and deterministic utility), an individual is considered "treated" if they choose not to purchase a particular product j, and the outcome of interest is whether they choose to purchase the substitute product k. This allows for a definition of diversion ratios as the fraction of treated individuals who choose the substitute.

We then show how a wide variety of results from the treatment-effects literature can be applied to interpret diversion ratios. Our main theoretical result derives the so-called LATE Theorem of Imbens and Angrist (1994) in the discrete-choice diversion ratio setting. We show that a *ceteris paribus* change in the price of good j identifies a local average of diversion ratios among a group of "compliers," or individuals who used to purchase good j at the original price but no longer do so at the higher price. We then apply several related results from the LATE toolkit. First, we show that although diversion ratios are defined in terms of small price changes, any product characteristic satisfying a monotonicity requirement can be used to construct an estimate of a diversion ratio. This makes it possible to identify and estimate diversion ratios in environments where quality (or distance from the consumer, etc.) varies but prices do not. Second, we show that second-choice data pertain to a different treatment-effects parameter, for which all individuals who were previously untreated receive treatment (not purchasing j). This is particularly relevant because it shows how diversion ratios are related to out-of-stock events (Conlon and Mortimer, 2013) and the second-choice survey data already being used by antitrust authorities. A 2017 commentary on retail mergers released by the UK Competition and Markets Authority (CMA) describes their use of diversion ratios for screening and analyzing mergers, saying:

> *Diversion ratios can be calculated in a number of different ways, depending on the information available in a particular case. In retail mergers, the CMA has most often used the results of consumer surveys to calculate diversion ratios. The diversion ratio attempts to capture what customers would do in response to an increase in prices. However, it can be difficult to survey a sufficiently large number of customers who would switch in response to a price rise to estimate a robust diversion ratio. Therefore, the CMA asks customers what they would do in response to the closure of a store (or stores).* (See Competition and Markets Authority (2017).)

We further analyze diversion ratios using the marginal-treatment-effects (MTE) framework of Heckman and Vytlacil (2005). We decompose average diversion ratios into individual diversion ratios (which we show are policy-invariant conditional on covariates), and weights on individuals (which are independent of the substitute product k). We show that the former do not depend on whether prices, quality, or availability is modified, whereas the latter do.

This decomposition has several useful applications. One (obvious) implication is that average diversion ratios are bounded by the range of underlying individual diversion ratios. A second implication is that assuming constant diversion ratios imposes strong parametric restrictions on the nature of demand and utility. Only two models of demand exhibit constant diversion: linear demand, and the plain logit model. Thus, treating diversion ratios as constant is akin to assuming that demand is well described by one of these two models. The third implication is that we can derive explicit formulas for the weights on individuals under the workhorse random-coefficients logit model. In principle, this allows us to show precisely how second-choice data, small price changes, and changes in other product characteristics trace out different subpopulations, and how the average diversion ratios measured with these interventions relate to one another. In principle, this allows us to answer questions like: "If I am interested in the diversion ratio from a small price change, how similar would second-choice survey data be?" or "If I only have diversion ratios measured from short-run variation in quality, how informative are they about diversion in response to a product removal ?"

We also show how diversion to the outside good D_{j0} is related to our notion of consumer welfare in random-coefficients logit-type models. We provide results linking willingness-to-pay (WTP) calculations that are commonplace in models of Nash-in-Nash bargaining (and hospital mergers in particular) to the question: "How many consumers would switch to the outside option if product j were no longer available?" We show that for the logit class of models these WTP calculations can be simplified even further as a function of individual own-share, s_{ij}, and a set of weights. This enables welfare analysis using WTP calculations, even in settings lacking an outside option (as is common in hospital demand).

We highlight the properties laid out above using the well-known examples of the Berry et al. (1999) automobile data and the Nevo (2000b) (fake) cereal data. Our empirical examples examine the role of diversion to best substitutes and to the outside good. We show that capturing both kinds of diversion ratios is important, and can at times be tricky. We also illustrate that different interventions (price changes, quality changes, and product removals) produce different weighting schemes over individual diversion ratios that can at times be quite different. For example, in the BLP automobile data, second-choice diversion understates substitution to the outside good when compared to small price changes by around 30%. Moreover, small changes in parametric specifications can lead to large changes in outside-good diversion (as well as markups and welfare) even when own-price elasticities appear stable. Other commonly-employed shortcuts, such as estimating a nested-logit with all products in a single nest, do a good job recovering the outside-good diversion, but predictably understate substitution to the closest substitutes.

Taken in total, we recommend that both academic researchers and antitrust practitioners pay careful attention to diversion to the outside good and to close substitutes, even when estimating parametric models of demand. Furthermore, we urge researchers to be aware that the diversion measure of interest need not coincide with the measure that is easily estimated from readily-available data (such as consumer switching surveys, etc.), although these extra data sources, when carefully incorporated, may substantially improve estimates of parametric demand models.

Related Literature

An ancillary goal of the article is to bring together two literatures – the applied theoretical literature that motivates the use of diversion for understanding merger impacts, and an applied econometric literature that articulates estimation challenges in settings for which the treatment effect of a policy can vary across individuals and may be measured with error.

By exploring the assumptions required for a credible (quasi)-experimental method of measuring diversion, we connect directly to the theoretical literature discussing the use and measurement of the diversion ratio.[3] Farrell and Shapiro (2010) suggest that firms themselves may track diversion in their 'normal course of business,' or that diversion ratios may be uncovered in Hart-Scott-Rodino filings. Hausman (2010) argues that the only acceptable way to measure a diversion ratio is as the output from a structural demand system. Reynolds and Walters (2008) examine the use of stated-preference consumer surveys in the UK for measuring diversion.

In spirit, our approach is similar to Angrist et al. (2000), which shows how a cost shock can

[3] The focus on measuring substitution away from product j (using second-choice data or stock-outs), rather than on the direct effect of a proposed merger, is more in line with the public finance literature on sufficient statistics (e.g., Chetty (2009)).

identify a particular local average treatment effect (LATE) for the price elasticity in a single-product setting. That approach does not extend to a differentiated-products setting because the requisite monotonicity condition may no longer be satisfied. Our results demonstrate how a *ceteris paribus* price change can identify substitution patterns in a multi-product setting. We also highlight the economic content of (even partial) second-choice data, which have been found to be valuable in the literature on structural demand estimation (e.g., Berry et al. (2004)).

The article proceeds as follows. Section 2 introduces the diversion ratio as an economic object and discusses its better-known cousin, the cross-price elasticity. We establish our theoretical results linking diversion ratios to treatment effects in Section 3. We highlight the theoretical properties of diversion ratios with empirical examples using data on autos from Berry et al. (1999) and (fake) data on cereal from Nevo (2000b) in Section 4. Section 5 concludes.

2. What is a diversion ratio?

We begin by considering a Bertrand-Nash game in which a multiproduct firm f sets the prices of products $j \in \mathcal{J}_f$ in order to maximize profits. For prices, we use P_j to denote the generic argument and p_j to denote a specific realization. The firm faces marginal costs c_j and demand curve $q_j(\mathbf{p})$ (where $\mathbf{p}$ is the vector of prices for all goods) and chooses price p_j to solve: $\arg\max_{\mathbf{p_f} \in \mathcal{J}_f} \pi_f(\mathbf{p}) = \sum_{j \in \mathcal{J}_f} (p_j - c_j) \cdot q_j(\mathbf{p})$. The first-order condition for each product j is given by:[4]

$$\rightarrow p_j = q_j(\mathbf{p}) \left[-\frac{\partial q_j}{\partial P_j}(\mathbf{p}) \right]^{-1} + c_j + \sum_{k \in \mathcal{J}_f \setminus j} (p_k - c_k) \underbrace{\frac{\partial q_k}{\partial P_j}(\mathbf{p}) \left[-\frac{\partial q_j}{\partial P_j}(\mathbf{p}) \right]^{-1}}_{D_{jk}(\mathbf{p})}$$

$$p_j(p_{-j}) = \underbrace{\frac{1}{1 + 1/\epsilon_{jj}(\mathbf{p})}}_{\text{Markup}} \left[c_j + \sum_{k \in \mathcal{J}_f \setminus j} (p_k - c_k) \cdot D_{jk}(\mathbf{p}) \right]. \quad (1)$$

Equation (1), shows the usual inverse elasticity ϵ_{jj} (Lerner) markup applied to the marginal cost of j. In multi-product pricing problems this term is augmented by the opportunity cost of selling other products $\sum_{k \in \mathcal{J}_f \setminus j} (p_k - c_k) \cdot D_{jk}(\mathbf{p})$ where the opportunity cost depends on the fraction of consumers who leave j for the substitute k (diversion ratio) and the price-cost margins for k. One way to think about what a merger does is that it raises the opportunity cost of selling j, by incorporating diversion (and price-cost margins) to additional products. This is the central idea in Farrell and Shapiro (2010), and is the reason that diversion features so heavily in the 2010 Horizontal Merger Guidelines.

There is some controversy about whether one should evaluate a merger by solving the system of first-order conditions (FOCs) defined by (1), as in Nevo (2000a), or by measuring how the merger shifts the FOC for each product j in isolation. (See, for example, Hausman (2010), Carlton (2010), Schmalensee (2009), Willig (2011), Carlton and Israel (2010), Gotts (2010), and Farrell and Shapiro (2010).) A common metric in the latter case is Upward Pricing Pressure (UPP). This measures how the term in brackets from (1) changes when f and g merge:

$$UPP_j = \underbrace{\Delta c_j}_{\text{efficiency}} + \sum_{k \in \mathcal{J}_g} \underbrace{(p_k - c_k) \cdot D_{jk}(p_j, p_{-j})}_{\Delta \text{opp cost}} \quad (2)$$

[4]It is common to write the denominator as the absolute value $\left| \frac{\partial q_j}{\partial P_j} \right| = -\frac{\partial q_j}{\partial P_j}$. These are equal so long as demand slopes down $\frac{\partial q_j}{\partial P_j} < 0$.

UPP trades off potential marginal-cost reductions (efficiency gains) against the increased opportunity cost that arises from diversion to additional products owned by firm g and their respective price-cost margins. Only the sign of UPP is directly interpretable, as it measures the effective change in marginal cost but not price.[5,6]

Whether one does full-merger simulation like Nevo (2000a) or simply calculates UPP, diversion ratios are a key feature in the first-order conditions of multi-product firms. In fact, for any study of multi-product firm behavior, the diversion ratio is one of the most important deliverables of any demand system. Later, in Section 3, we show how the WTP measure, commonly used in evaluating hospital mergers and as an input into Nash-in-Nash bargaining models, is related to diversion to the outside good. (See Town and Vistnes (2001), Capps et al. (2003), and (Crawford and Yurukoglu, 2012).) In related work, Katz and Shapiro (2003) show that the "aggregate diversion ratio," or total diversion to all products *except* the outside option, is related to the SSNIP (small but significant and non-transitory increase in price) and hypothetical monopolist tests that antitrust authorities use to define relevant markets.

How is diversion different from cross-price elasticity?

The diversion ratio can be written as the ratio of cross- and own-price elasticities multiplied by the respective ratio of market shares.[7] Unlike own-price elasticities, for which there is a natural relationship between the value of the elasticity and the market power of the firm (Lerner Markup), cross-price elasticities alone are insufficient to calculate a compensating

[5] Jaffe and Weyl (2013) estimates a pass-through rate to map opportunity-cost effects of a merger into price effects. Miller et al. (2016) and Cheung (2011) find that the price effects of a merger, and errors in predicting these effects, depend on the nature of competition among non-merging firms.

[6] Merger analyses often compute the "compensating marginal-cost reduction," which is the marginal-cost reduction necessary to restore pre-merger prices. See Werden (1996).

[7] Recall that the cross-price elasticity is defined as: $\epsilon_{jk} = \frac{\partial q_k}{\partial P_j}\frac{p_j}{q_k}$ and that the ratio of cross- and own-price elasticities yields: $\frac{\epsilon_{jk}}{\epsilon_{jj}} = \frac{\frac{\partial q_k}{\partial P_j}}{\frac{\partial q_j}{\partial P_j}}\frac{q_j}{q_k} = -D_{jk}\frac{s_j}{s_k}$.

marginal-cost reduction. Moreover, larger cross-price elasticities may not even signify closer substitutes.[8] Finally, note that there may be cases where the diversion ratio is identified, but the constituent parts $\frac{\partial q_k}{\partial P_j}, \frac{\partial q_j}{\partial P_j}$ or $\epsilon_{jk}, \epsilon_{jj}$ are not separately identified. Indeed, we highlight several hypothetical experiments for which the diversion ratio (or some average of diversion ratios) can be recovered even when the underlying demand curves or consumer utility functions cannot.[9]

The remainder of the article focuses on two key aspects of diversion ratios: (1) what a *ceteris paribus* change in price or availability can tell us about diversion ratios; (2) what restrictions are imposed on diversion ratios by parametric models of demand commonly used by researchers and practitioners.

3. A Treatment-Effects Interpretation of Diversion Ratios

The Wald (1940) Estimator

Diversion ratios provide a way to answer the question: "What can a price change tell us about substitution patterns?" We begin by considering a *ceteris paribus* increase in the price of good j from $p_j \to p'_j$ and define the corresponding Wald estimator, which compares the ratio of the change in sales of k to that of j.[10] The Wald estimator has the advantage that it is often readily observed from the data for a particular price change:

$$\text{Wald}(p_j, p'_j, x) = \frac{q_k(p'_j, x) - q_k(p_j, x)}{-\left(q_j(p'_j, x) - q_j(p_j, x)\right)}. \tag{3}$$

[8] Consider two substitutes: the first has a cross-price elasticity with j of $\epsilon_{jk} = 0.03$ and $s_k = 0.1$ whereas the second has a cross-price elasticity of $\epsilon_{jk'} = 0.01$ and $s_{k'} = 0.35$. More consumers switch to k' than to k, even though k has a larger cross-price elasticity.

[9] The UK CMA survey asking shoppers: "If this store were closed where would you shop?" is a prime example. We could estimate a particular average diversion ratio, but would have no ability to estimate own- or cross-price elasticities.

[10] Implicitly, x contains all other relevant characteristics including: prices of other goods p_{-j}, product quality and assortment, etc. That p'_j represents a price increase is arbitrary. We could simply exchange (p_j, p'_j) and consider a price decrease.

The diversion ratio $D_{jk}(p_j, x) \equiv \frac{\partial q_k}{\partial P_j}(p_j, x) / - \frac{\partial q_j}{\partial P_j}(p_j, x)$ can be obtained as the limit of the Wald estimator in (3) where the price increase (or decrease) becomes small, so long as demand slopes *strictly* downwards $\frac{\partial q_j}{\partial P_j} < 0$:[11]

$$\lim_{p'_j \to p_j} \frac{q_k(p'_j, x) - q_k(p_j, x)}{-(q_j(p'_j, x) - q_j(p_j, x))} \to \frac{\frac{\partial q_k}{\partial P_j}(p_j, x)}{-\frac{\partial q_j}{\partial P_j}(p_j, x)} \equiv D_{jk}(p_j, x) \tag{4}$$

A large literature establishes the connection between Wald estimators like (3) and local average treatment effects (LATE).

Discrete Choice and a LATE Theorem

An easy way to see the connection between the Wald estimator and LATE is to assume that the demand functions in (4) are derived from a discrete-choice model of demand, where consumer i has unit demand and faces prices (p_j, p_{-j}). Consumer i selects product j from set $\mathcal{J}$ (which includes an outside or no-purchase option) in order to maximize utility. Define d_{ij} as an indicator for consumer i choosing product j:[12]

$$d_{ij}(p_j, x) = \begin{cases} 1 & u_{ij}(p_j, x) > u_{ij'}(p_j, x) \text{ for all } j' \in \mathcal{J} \text{ and } j' \neq j. \\ 0 & o.w. \end{cases}$$

To obtain market demand curves and diversion ratios, one must integrate over the distribution of individual utilities u_{ij}, which we denote F_i. When utility is deterministic, this amounts to a simple average. Alternatively, one could consider an additive random utility model (ARUM), so that $u_{ij} = V_{ij}(x) + \varepsilon_{ij}$. In this case, an integral (or expectation) over the distribution of heterogeneous individuals F_i may involve both the observable component

[11] This is an application of L'Hôpital's Rule and also requires that q_k is differentiable (locally) about (p_j, x).

[12] This setup includes most random utility models (RUMs), including logit and logit variants. It also encompasses a broad class of both random and deterministic choice models, including a variety of behavioral models where consumers make "mistakes" or fail to consider all available products.

$V_{ij}(x)$ and/or the unobservable component ε_{ij}.

In this multiple discrete-choice setting, we derive an equivalent to the LATE theorem of Imbens and Angrist (1994). Our main theoretical result shows that under some relatively weak discrete-choice assumptions, a price change can be used to identify the average diversion ratio among a particular set of consumers (compliers).

Proposition 1 (Analogue to LATE Theorem (Imbens and Angrist, 1994))**.**

Under the following conditions:

(a) Mutually Exclusive and Exhaustive Discrete Choice: $d_{ij} \in \{0,1\}$ *and* $\sum_{j\in\mathcal{J}} d_{ij} = 1$*;*

(b) Exclusion: $u_{ik}(p_j, x) = u_{ik}(p'_j, x)$ *for all* $k \neq j$ *and any* (p_j, p'_j)*;*

(c) Monotonicity: $u_{ij}(p'_j, x) \leq u_{ij}(p_j, x)$ *for all* i *and any* $(p'_j > p_j)$*;*

(d) Existence of a first-stage: $d_{ij}(p_j, x) = 1$ *and* $d_{ij}(p'_j, x) = 0$ *for* $(p'_j > p_j)$ *for some* i*; and*

(e) Random Assignment: $(u_{ij}(P_j, x), u_{ik}(P_j, x)) \perp P_j$*.*

then the Wald estimator from (3)*:*

$$\frac{q_k(p'_j, x) - q_k(p_j, x)}{-\left(q_j(p'_j, x) - q_j(p_j, x)\right)} = \mathbb{E}[D_{jk,i}(x) | d_{ij}(p_j, x) > d_{ij}(p'_j, x)]$$

where the expectation $\mathbb{E}(\cdot)$ *is taken over the (heterogeneous) individuals* F_i *in the population. (Proof in Appendix A.1.)*

In words, the proposition says that the Wald estimator identifies the average diversion ratio among "compliers," or those individuals who would buy j at p_j but no longer buy j at p'_j (holding x fixed). In other words, the treatment is defined as "not buying j" and compliance types are defined in Section 5. The outcome is the event that consumer i purchases product k: $d_{ik}(P_j, x) = 1$, and the price P_j functions as the "instrument" because it monotonically increases the probability of treatment. The important, but perhaps obvious implication of Proposition 1 is that the individual diversion ratios $D_{jk,i}(x)$ do not depend on the price of j because they are determined by how consumers rank substitutes k and k'. The price of j does, however, determine which individuals are "treated" as P_j rises. The bottom panel of Section 5 reports the definitions of various treatment-effects parameters for the diversion ratio. We note that except under very specific circumstances, these treatment-effects estimates (ATE, ATT, ATUT, LATE) will not agree with one another.

The most common threat to the LATE approach is the presence of "defiers" who must be ruled out by the monotonicity assumption. That is less of a concern here, because monotonicity is guaranteed by the "Law of Demand." A second common challenge is the plausibility of the exclusion restriction. In the case of diversion, this requires that utility for k is unaffected by the price of j, which is an uncontroversial feature of most discrete-choice utility models.[13]

In the case of diversion, the controversial assumption in Proposition 1 is likely to be (e) Random Assignment, which enables the price P_j to function as an "instrument." Assumption (e) implies that the distribution of consumer utilities *as a function of price* cannot differ with the observed realizations of prices (p_j, p_j'). This is the usual *simultaneity problem* in demand and supply that dates back to Working (1927), and would obviously be violated if an unobservable demand shock is correlated with both prices and consumer preferences as in Berry (1994). This is also violated if there is selection that leads to less-price-sensitive consumers facing higher prices in the data (e.g., in the presence of price discrimination).

We caution that there is no "free lunch" in this case, and that LATE estimates of diversion are not absolved from concerns about price endogeneity. At the same time, assumption (e) does not present a problem for *interpreting* diversion ratios as treatment effects when we analyze a *ceteris paribus* change in price, in the same way that analyzing the price-elasticity of a demand curve is not complicated by the problems associated with endogeneity of price that arise in estimation of that elasticity.

[13] A potential violation might be a behavioral model with framing effects such as preferences for the "second cheapest" product.

Treatment Effect Heterogeneity

Different consumers are likely to exhibit different diversion ratios. We show how to decompose a LATE measure into (i) an underlying heterogeneous distribution and (ii) a set of weights, in a way that mirrors Heckman and Vytlacil (2005). We begin by re-writing the Wald estimator:[14]

$$\text{Wald}(p_j, p'_j, x) = \int_{p_j}^{p'_j} D_{jk}(P_j, x) w(P_j)\, \partial P_j \text{ with } w(P_j) = \frac{\frac{\partial q_j(P_j,x)}{\partial P_j}}{\int_{p_j}^{p'_j} \frac{\partial q_j(P_s,x)}{\partial P_s} \partial P_s} \tag{5}$$

This tells us that the average diversion ratio measured is just the weighted average of diversion ratios at each $D_{jk}(P_j, x)$, where the weights correspond to the consumers who leave j at each price P_j as a fraction of all consumers who leave j as the price increases from $(p_j \to p'_j)$. All things being equal, as demand for j becomes more elastic, this will concentrate more of the weight $w(P_j)$ towards the diversion ratios measured at prices closer to p_j. As demand becomes less elastic, the weight is spread more evenly across price increments.

We can further decompose the heterogeneity by observing that at each P_j, $D_{jk}(P_j, x)$ implicitly integrates over a heterogeneous distribution of individuals as illustrated in Propo-

[14] Observe that $D_{jk}(P_j, x) = \frac{\frac{\partial q_k(P_j,x)}{\partial P_j}}{\left|\frac{\partial q_j(P_j,x)}{\partial P_j}\right|}$ so that $D_{jk}(P_j, x)\left|\frac{\partial q_j(P_j,x)}{\partial P_j}\right| = \frac{\partial q_k(P_j,x)}{\partial P_j}$. Also notice that the integral in the denominator of the weights is just the change in sales of j: $\int_{p_j}^{p'_j} \frac{\partial q_j(P_j,x)}{\partial P_j} \partial P_j = -\left(q_j(p'_j, x) - q_j(p_j, x)\right) = q_j(p_j, x) - q_j(p'_j, x)$.

sition 1:[15,16]

$$
\begin{aligned}
(5) &= \int_{p_j}^{p_j'} \int D_{jk,i}(P_j, x) w_i(P_j, x)\, \partial P_j\, \partial F_i \quad \text{ with } w_i(P_j, x) = \frac{\left| \frac{\partial q_{ij}(P_j, x)}{\partial P_j} \right|}{q_j(p_j, x) - q_j(p_j', x)} \\
&= \int D_{jk,i}(x)\, w_i(p_j, p_j', x)\, \partial F_i \quad \text{ with } w_i(p_j, p_j', x) = \frac{q_{ij}(p_j', x) - q_{ij}(p_j, x)}{q_j(p_j, x) - q_j(p_j', x)} \qquad (6)
\end{aligned}
$$

We've now provided two alternatives for decomposing heterogeneity in diversion ratios. In (5), we show that the average diversion ratio varies with price, and we show how the LATE/Wald estimator weights diversion ratios at different price increments. In (6), we show that the LATE/Wald estimator can be decomposed into diversion ratios of individual consumers (which are now independent of price P_j) and a set of individual-specific weights $w_i(p_j, p_j', x)$ (which depend on realizations of prices (p_j, p_j') but not the substitute k). These weights determine the (conditional) probability than an individual of type i is a complier under a particular intervention (p_j, p_j').

We know that the different "instruments" measure different LATEs by tracing out different groups of compliers. This means that we can construct a local average measure of diversion ratios with respect to other characteristics. For example, consider a *ceteris paribus* reduction in quality from $\xi_j \rightarrow \xi_j'$, or an arbitrary characteristic $z_j \rightarrow z_j'$.[17] This allows us to re-write (6) as:[18]

$$
\text{Wald}(z_j, z_j', x) = \int D_{jk,i}(x)\, w_i(z_j, z_j', x)\, \partial F_i \quad \text{ with } w_i(z_j, z_j', x) = \frac{q_{ij}(z_j', x) - q_{ij}(z_j, x)}{q_j(z_j, x) - q_j(z_j', x)}. \qquad (7)
$$

[15]This requires that $D_{jk,i}(x) = D_{jk,i}(P_j, x)$ for any p_s, which follows from the proof of Proposition 1. For random-utility discrete-choice models this implies: $Pr(u_{ik} > \max_{k' \in \mathcal{J} \setminus \{j,k\}} u_{ik'}) \perp P_j$, which is guaranteed by the exclusion restriction and random assignment.

[16]The second integral arises because at each p_s, one can express $D_{jk}(P_j, x) = \int D_{jk,i}(P_j, x)\, \partial F_i$ as the integral over heterogeneous individuals. To exchange the order of integration we use Fubini's Theorem, which applies because $D_{jk}(P_j, x) \geq 0$ and $w_i(P_j, x) \geq 0$ everywhere.

[17]Berry and Haile (2014) exploit the idea that quality functions as "minus price."

[18]When discussing quality (or other instruments) we treat prices p_j as part of the fixed characteristics x.

Equation (7) exploits the fact that the individual-specific diversion ratios don't vary with price (or even rely on a price change), but rather that the weights vary when we use quality rather than price as the instrument. Any product characteristic z_j is valid, so long as it satisfies the monotonicity condition $u_{ij}(z'_j) \leq u_{ij}(z_j)$ for all i and any $(z'_j > z_j)$.[19] This may seem surprising at first, because we might expect cars to have one elasticity with respect to price, and a different elasticity with respect to fuel economy. What (6) says is that at the individual level (and conditional on x) there is only one diversion ratio and it is a structural parameter that is invariant to whether we change price, quality, or availability. The choice of, or variation in, the instrument simply determines the weights applied to the individual diversion ratios.

This is consistent with results in Heckman (2010), Heckman and Vytlacil (2005),and Carneiro et al. (2011) on Marginal Treatment Effects (MTE). Any treatment-effects parameter from Section 5 (LATE, ATE, ATT, ATUT, etc.) can be written as the integral of individual marginal treatment effects ($D_{jk,i}(x)$ in our case) over some set of weights. As an example, the returns to college attendance at the individual level are fixed and (assumed to be) policy invariant. However, different policy interventions (instruments) such as merit scholarships, affirmative action, or financial aid induce different students to attend college (different compliers) and thus weight the underlying individual treatment effects differently, producing different average effects.

Special Case: Second-Choice Data

An important special case is that of second-choice data (i.e., cases in which all buyers i of product j are treated, such that $d_{ij}(z_j, x) = 1$ and $d_{ij}(z'_j, x) = 0$). In practice, this could be accomplished in one of several ways: (a) by setting p_j equal to the *choke price* $\overline{p}_j$ such

[19] In a random-coefficients logit model, this would require that the sign of β_i on z_j is the same for all individuals. For example, this would rule out cases where some consumers prefer "Mushy" cereal and others do not.

that $q_j(\overline{p}_j, x) = 0$; (b) by reducing the quality (or some other characteristic) such that no consumers choose j: $q_j(z'_j, x) = 0$; or (c) by eliminating the product j from the choice set, $\mathcal{J} \setminus j$. In this case, Proposition 1 implies that:

$$\text{Wald}(p_j, \overline{p}_j, x) = \frac{E[d_{ik}(\overline{p}_j, x)] - E[d_{ik}(p_j, x)]}{E[d_{ij}(p_j, x)] - \underbrace{E[d_{ij}(\overline{p}_j, x)]}_{=0}} = E[D_{jk,i}(x) | d_{ij}(p_j, x) = 1] = \text{ATUT}. \quad (8)$$

We measure the average diversion ratio for all individuals who would have bought j at the original price p_j. Since our treatment indicator corresponds to "not buying j," this is the ATUT (average treatment on the untreated) and is equivalent to the gain in sales by k divided by the lost (original) sales of j at p_j.[20,21] The instrument we use to obtain second-choice data is irrelevant as all types of second-choice data identify the ATUT.[22]

One can now ask whether the UK CMA's question "If this store were to close, where would you shop instead?" is useful for the purpose of applying antitrust policy. The answer depends on whether the ATUT $E[D_{jk,i}(x) | d_{ij}(p_j, x) = 1]$ is a good estimate for $D_{jk}(P_j, x)$ at the value of P_j that is of interest to the CMA (potentially the pre-merger price). This also provides a specific interpretation for the substitution measured by natural-disaster-induced hospital closures in Raval et al. (2019).

In addition, our framework sheds light on the value of second-choice data in parametric models of demand such as Berry et al. (2004) (microBLP). In microBLP, the authors have access to survey data on consumers' second choices. They report finding extra moments from this second-choice data useful in estimating nonlinear (substitution) parameters. Our expression in (8) explains why: these second choices are informative about the average

[20]The ATT at p_j is just $\frac{s_k(p_j, x)}{1 - s_j(p_j, x)}$. That is, the share of people purchasing k among all non-j buyers at p_j is the observed share because being treated is "not buying j." As we demonstrate later, the plain 'IIA' logit exhibits constant treatment effects and thus has the property that the ATE=ATT=ATUT.

[21]The ATT may look familiar as the rate of substitution from the plain 'IIA' logit, although no parametric assumptions have been imposed beyond those in Proposition 1.

[22]Note that the w_i's are equal at the "choke price" and "choke quality" in (6) and (7): $w_{ij} = \frac{q_{ij}(z_j, x)}{q_j(z_j, x)}$.

diversion ratios of those surveyed. If the survey is a random sample of buyers of j, then it is informative about the ATUT, and "micro-moments" constrain the ATUT of the parametric model to match the ATUT from the survey. Additional information on average diversion ratios may be particularly useful for researchers if the corresponding counterfactuals (such as prospective merger evaluation, recovery of implied price-cost margins, or WTP calculations) depend on accurate measures of diversion.

Differences Among Treatment-Effects Parameters

Researchers often want to know whether or not a LATE/Wald estimator is a good estimate of the ATT, ATE, ATUT, or the marginal effect at some x. In a typical problem, the answer depends on heterogeneity in the individual treatment effects ($D_{jk,i}(x)$ in our case) and selection into treatment.[23] Consider a second-order Taylor expansion of q_k around p_j, where we define $\Delta p_j = p'_j - p_j$:

$$\frac{q_k(p'_j, x) - q_k(p_j, x)}{\Delta p_j} \approx \frac{\partial q_k(p_j, x)}{\partial P_j} + \frac{\partial^2 q_k(p_j, x)}{\partial P_j^2} \Delta p_j + O(\Delta p_j)^2. \tag{9}$$

The difference between the LATE/Wald estimator and its limit as $\Delta p_j \to 0$, defined in (4) as $D_{jk}(p_j, x)$, is given by:

$$\underbrace{\frac{q_k(p'_j, x) - q_k(p_j, x)}{q_j(p_j, x) - q_j(p'_j, x)}}_{=\text{Wald}(p_j, p'_j, x)} - D_{jk}(p_j, x) \approx -\frac{D_{jk}(p_j, x)\frac{\partial^2 q_j}{\partial P_j^2} + \frac{\partial^2 q_k}{\partial P_j^2}}{\frac{\partial q_j}{\partial P_j} + \frac{\partial^2 q_j}{\partial P_j^2}\Delta p_j} \Delta p_j \tag{10}$$

As one might expect, as Δp_j becomes larger, the difference between the LATE/Wald(p_j, p'_j, x) estimator and $D_{jk}(p_j, x)$ grows. As p'_j increases, we treat more individuals (and thus reduce the variance of our Wald estimator) but we begin to average over diversion ratios at prices

[23] For example, when asking whether the wage effect on the average college attendee is similar to that of the marginal college attendee, the answer depends on the unobserved underlying distribution of "ability," among other things.

further away from p_j. This has the effect of changing the weights in (6). Diversion is unique vis-a-vis other treatment-effects applications because it links differences in measures of diversion ratios to the underlying properties of demand; specifically to the curvature of demand, which enters the numerator of (10).[24] This makes it easy to sign the difference or derive bounds for well-known demand forms. We discuss several common parametric forms (linear, log-linear, logit, nested logit, and mixed logit) in Appendix A.2.

For some treatment-effects parameters, under the discrete-choice assumption (but without additional parametric assumptions beyond those in Proposition 1) we can derive analytic expressions. One example that does not depend on the parametric form is the $ATT = E[D_{jk,i}(x)|d_{ij}(p_j, x) = 0] = \frac{s_k(p_j,x)}{1-s_j(p_j,x)}$. This is just the share of consumers choosing k as a fraction of all those not choosing j. In the case of second-choice data ($ATUT$), everyone who purchased j at p_j is treated, and we weight all individuals by $w_i(p_j, x) = s_{ij}(p_j, x)$ (their initial purchase probability for j), although we cannot say anything at all about $D_{jk,i}(x)$ without additional assumptions.[25] The ATE (for which everyone is treated) is $ATE = E[D_{jk,i}(x)] = (1 - s_j) \cdot ATT + s_j \cdot ATUT$.[26] We expect most practitioners to be interested in the properties of small price changes under the LATE/Wald estimator and its limit $D_{jk}(P_j, x)$, or second-choice data (ATUT), so we focus on these measures.

Special Case: Constant Treatment Effects (Diversion Ratios)

There are some special models for which all of the treatment-effects measures coincide, and thus, whether one considers a small price (or quality) change or uses second-choice data, one obtains the same diversion ratio. These are cases where the model exhibits *constant*

[24] Also notice that we do not need to assume discrete-choice demand for this result.

[25] This provides an explanation for the perceived practice of antitrust agencies, absent better data, of assuming that diversion ratios are proportional to pre-existing market shares. This is equivalent to assuming that the ATT is a good estimate of the ATUT (which more generally implies no selection).

[26] The expression is derived from $ATE = Pr(T_i = 0) \cdot ATUT + Pr(T_i = 1) \cdot ATT$ where the probability of being treated (not buying j) is just $1 - s_{ij}(p_j, x)$. Of course $(1 - s_j) \times ATT = \frac{s_k}{1-s_j} \times (1 - s_j) = s_k$ which is just the share of k and exactly what one would expect.

treatment effects and all individuals have identical diversion ratios.

Two examples for which the numerator of (10) is equal to zero, and that produce constant treatment effects and diversion ratios, are: the linear model, for which $\frac{\partial^2 q_k}{\partial P_j^2} = 0$ for all (j, k), and the logit model, which sets the numerator of (10) to zero: $D_{jk} = -\frac{\partial^2 q_k}{\partial P_j^2} / \frac{\partial^2 q_j}{\partial P_j^2}$.[27] This has implications in both directions. First, if we assume linear or logit demand, we are implicitly assuming that diversion ratios do not vary with price or across individuals. Second, if we treat the diversion ratio as if it were constant, we are implicitly restricting the true demand system to be consistent with linear demand or plain 'IIA' logit. Researchers should be just as nervous about these restrictions as they would be about constant treatment effects in other contexts, such as assuming homogeneous returns to college attendance.

The nested-logit model may seem like it has constant diversion ratios (as the product of two plain 'IIA' logits), but in fact does not, which we show in Appendix A.2. This means that the diversion ratio from a small change in a product characteristic (including price) is not the same as a diversion ratio obtained from second-choice data (ATUT); we have confirmed this fact in simulations. To illustrate this, note that not all "individuals" have the same diversion ratios, and that diversion ratios vary with the category of the initial product choice. There are different diversion ratios (for small changes in the "index") for products in the same nest, $\frac{s_{k|g} Z(\sigma, s_g)}{1 - s_{j|g} Z(\sigma, s_g)}$, and products in separate nests, $\frac{s_k (1-\sigma)}{1 - s_{j|g} \cdot Z(\sigma, s_{g(j)})}$, where $Z(\sigma, s_g) = [\sigma + (1-\sigma) s_g]$.

Application to Random-Coefficients Logit

The results in the previous sections can also be applied to the workhorse random-coefficients logit model that is popular in industrial organization and widely used for merger evaluation in particular (e.g., (Berry et al., 1995) and (Nevo, 2001)). The random-coefficients logit

[27] This is derived in Appendix A.2. An easier way to see constant diversion under the IIA logit is to observe that $D_{jk,i} = \frac{s_{ik}}{1 - s_{ij}} = \frac{s_k}{1 - s_j}$ for all individuals. This differs from diversion under linear demand. As is shown in Jaffe and Weyl (2010), these measures are allowed to disagree.

model parametrizes utility as:

$$u_{ij} = \beta_i z_j - \alpha_i P_j + \xi_j + \varepsilon_{ij}, \text{ with } \varepsilon_{ij} \sim \text{IID Type 1 EV} \tag{11}$$

where $f(\beta_i, \alpha_i; \theta)$ is a parametric distribution known up to a parameter θ.[28] This class of models has an "index" so that an "individual" (fixing α_i, β_i but not ε_i) does not discriminate among changes that make (z_j, p_j, ξ_j) worse so long as they change the index by an equal number of units. We exploit this index property when computing the integration weights $w_{ij}(x)$. The decomposition from (6) allows us to write any treatment-effects parameter for diversion ratios under the random-coefficients logit as:[29]

$$D_{jk}(x) = \int D_{jk,i}(x) w_{ij}(x) \partial F_i = \int \frac{s_{ik}(x)}{1 - s_{ij}(x)} w_{ij}(x) \partial F_i = \int s_{ik}(x) \, \widetilde{w}_{ij}(x) \partial F_i \tag{12}$$

We make use of several properties. First, when we integrate out over ε_i, each *individual's* diversion ratio follows a (different) logit such that $D_{jk,i}(x) = \frac{s_{ik}(x)}{1 - s_{ij}(x)}$.[30] Consistent with (6) and holding for any discrete-choice model satisfying the assumptions of Proposition 1, individual diversion ratios do not depend on P_j. This is because diversion already conditions on individuals *not buying good* j.[31] Whether one considers changes of prices, quality, or other characteristics, and whether those changes are large (including second-choice data) or small, only affect *weighting* and not individual diversion ratios. At the level of the individual i, diversion ratios depend only on how i ranks k relative to some alternative k'.

[28] The convention in Heckman and Vytlacil (2005) is to consider an "individual agent" as including the error term (here ε_i). Instead, we follow the random-coefficient logit convention and treat an "individual" as a (β_i, α_i) but integrating over, rather than conditioning on, ε_i.

[29] Observe that $\widetilde{w}_{ij}(x) = \frac{w_{ij}(x)}{1 - s_{ij}(x)}$. We abuse notation and suppress the (p'_j, p_j) or (z'_j, z_j) arguments because a variety of instruments (or none at all - as in second-choice data) could be used.

[30] The fact that individual demands follow plain 'IIA' logit is well-known, but a derivation is provided in Appendix A.2.

[31] Note: this is *not* the same thing as saying that $s_{ij}(x)$ or $s_{ik}(x)$ wouldn't be different at a different value of (p_j, p_k); they definitely would be!

The second property is that the diversion ratio is always bounded above by $\max_i \frac{s_{ik}(x)}{1-s_{ij}(x)}$. Thus, the average diversion ratio can never exceed the largest diversion ratio for any individual. In practice this is the individual for whom the combination of $s_{ik}(x)$ and $s_{ij}(x)$ is largest. Unless we see very large individual-choice probabilities $s_{ik}(x)$ we should not expect to see very large diversion ratios, even for highly similar products (such as red and blue buses).

The third property, which is specific to the logit family, allows us to rewrite (12) as the integral of two terms: i's share for the substitute $s_{ik}(x)$, and a re-defined weight $\widetilde{w}_{ij}(x) = \frac{w_{ij}(x)}{1-s_{ij}(x)}$, which depends on j. In Table 2, we compute the corresponding weights for (6) for various treatment-effects parameters and instruments. For expositional purposes, we focus on the right-hand side of (12) so that we are always integrating the individual's share for $k, s_{ik}(x)$, over the adjusted $\widetilde{w}_{ij}(x)$. We begin by considering a unit change in the index from (11), which we could accomplish by reducing ξ_j. In this case, all individuals are weighted proportional to their purchase share of the initial good $s_{ij}(x)$, which seems intuitive. If instead, we consider a unit change in a characteristic z_j or price p_j, we concentrate more weight on those individuals more sensitive to the characteristic, $\widetilde{w}_{ij}(x) \propto s_{ij}(x)|\alpha_i|$. If all individuals possess the same $\alpha_i = \alpha$ then this drops out and these are identical to weights on the marginal change in the quality index $s_{ij}(x)$. Second-choice data instead weights according to $\widetilde{w}_{ij}(x) \propto \frac{s_{ij}(x)}{1-s_{ij}(x)}$. Relative to a small change in the index, it places more weight on individuals with higher initial shares for $s_{ij}(x)$. However, in many cases (such as the BLP automobile example), individual purchase probabilities are small, so that the denominator $1 - s_{ij}(x) \approx 1$ and the second-choice weights are nearly identical to those from a marginal change in product quality.[32] When the share $s_{ij}(x)$ becomes large, the second-choice weights diverge from the marginal quality-change weights. In practice, this tends to require a small

[32]This is not the same thing as saying that s_j is small. There may still be "individuals" with a high purchase probability for j, even if the overall market share s_j is small.

outside-good share (at least for some individuals).

This decomposition highlights the role of random coefficients in diversion ratios. Random coefficients have two effects: the first is to increase dispersion in the share of the substitute $s_{ik}(x)$ (or underlying diversion ratios $\frac{s_{ik}(x)}{1-s_{ij}(x)}$), and the second is that when we consider diversion ratios estimated from changes in characteristics with random coefficients, it places more weight on the individuals most sensitive to that characteristic. Under the random-coefficients logit model, whether or not second-choice data (potentially from surveys) produces an estimate similar to that of a small price change depends on how much $s_{ij}(x)|\alpha_i|$ differs from $s_{ij}(x)$ and how correlated $s_{ik}(x)$ is with those weights. We illustrate these properties in our empirical examples.

Relationship to Willingness to Pay

In addition to UPP and merger simulation, another important object of analysis is the WTP measure of Town and Vistnes (2001) and Capps et al. (2003). WTP measures the value of including an option j in a consumer's choice set, and is an important input into the analysis of hospital-insurer networks (e.g., Ho (2006),Ho (2009), and Ericson and Starc (2015)) and Nash-in-Nash bargaining between hospitals and insurers (e.g., Gowrisankaran et al. (2015) and Ho and Lee (2019)). The WTP measure has also been used outside the hospital-insurer context in the analysis of cable bundles (e.g., Crawford and Yurukoglu (2012)).

We consider a simple version of the WTP measure under the same random-coefficients logit model as before. Consumer utility is given by (11), which we write as $u_{ij}(x) = V_{ij}(x) + \varepsilon_{ij}$. There are well-known results (Manski and McFadden, 1981), which show that the expectation of the maximum has a closed form: $E[\max_{k\in\mathcal{J}} u_{ik}(x)] = \log\sum_{k\in\mathcal{J}}\exp[V_{ik}(x)]$. If we follow Capps et al. (2003) or Ho (2006) and ask: "What is the value to consumer i of including j in the choice set?," we obtain:[33]

$$WTP_i(j) = E[\max_{k \in \mathcal{J}} u_{ik}(x)] - E[\max_{k' \in \mathcal{J} \setminus j} u_{ik'}(x)] = \log\left(\sum_{k \in \mathcal{J}} \exp[V_{ik}(x)]\right) - \log\left(\sum_{k \in \mathcal{J} \setminus j} \exp[V_{ik}(x)]\right) \tag{13}$$

We make use of the following relationships: (1) the individual outside-good choice probability is given by $s_{i0}(\mathcal{J}, x) = \frac{1}{\sum_{k \in \mathcal{J}} \exp[V_{ik}(x)]}$; (2) the outside-good choice probability after removing j increases by the individual share of j times the individual diversion ratio from j to the outside good, $s_{i0}(\mathcal{J} \setminus j, x) = s_{i0}(\mathcal{J}, x) + D_{j0,i}(x) \cdot s_{ij}(x)$; and (3) for members of the logit family, $D_{j0,i}(x) = \frac{s_{i0}(x)}{1 - s_{ij}(x)}$. This allows us to rewrite (13) as:[34]

$$\begin{aligned} &= \log\left(\frac{s_{i0}(\mathcal{J} \setminus j, x)}{s_{i0}(\mathcal{J}, x)}\right) = \log\left(1 + \frac{D_{j0,i}(x) s_{ij}(x)}{s_{i0}(\mathcal{J}, x)}\right) = \log\left(1 + \frac{s_{i0}(\mathcal{J}, x) \cdot s_{ij}(x)}{(1 - s_{ij}(x)) \cdot s_{i0}(\mathcal{J}, x)}\right) \\ &= \log\left(1 + \frac{s_{ij}(x)}{1 - s_{ij}(x)}\right) \approx \frac{s_{ij}(x)}{1 - s_{ij}(x)} \end{aligned} \tag{14}$$

At the individual level, the amount i would pay to have j in his or her choice set is related to the percentage change in the outside good share, or the diversion from j to the outside good $D_{j0,i}(x)$. This is intuitive: products that add the most value are those without close substitutes, which draw in consumers from the outside option. Perhaps surprising is that this expression simplifies so that $WTP_i(j)$ can be written solely as a function of the individual choice probability for j, $s_{ij}(x)$. This result is of practical relevance because many hospital network papers (e.g., Ho (2006, 2009) and Capps et al. (2003)) omit the outside good, and (14) shows us that $WTP_i(j)$ requires only an estimate of the individual choice probability $s_{ij}(x)$.

[33] We simplify by ignoring the probability that consumers face additional ex-ante risk over diagnoses. In this sense, our version more closely resembles Crawford and Yurukoglu (2012).

[34] The approximation arises from the first-order Taylor expansion: $\ln(1 + x) \approx x$. This approximation is valid when s_{ij} is small.

It is common to aggregate $WTP(i,j)$ across individuals. One convention is to follow Manski and McFadden (1981) and convert difference in utility to money-metric equivalent variation (EV) and aggregate across individuals:[35]

$$\overline{WTP}(j) = \int \frac{1}{|\alpha_i|} WTP_i(j) \partial F_i \approx \int \frac{s_{ij}(x)}{|\alpha_i|(1 - s_{ij}(x))} \partial F_i = \int D_{j0,i}(x) \frac{s_{ij}(x)}{|\alpha_i| \cdot s_{i0}(\mathcal{J}, x)} \partial F_i \tag{15}$$

The last expression in (15) results in the same form as (7), in which we integrate diversion from j to the outside good $\int D_{j0,i}(x) w_{ij}(x) \partial F_i$. The difference is that (15) uses a different set of weights $w_{ij}(x) = \frac{s_{ij}(x)}{|\alpha_i| \cdot s_{i0}(\mathcal{J},x)}$. Therefore, we include this as the final example of our decomposition in Table 2. The connection between second-choice data and WTP is that measures of diversion that use second-choice data integrate $s_{ik}(x)$ over $\widetilde{w}_{ij}(x) = \frac{s_{ij}(x)}{1-s_{ij}(x)}$, whereas in the absence of heterogeneous tastes for price, WTP integrates a constant 1 over the same set of weights $\widetilde{w}_{ij}(x) = \frac{s_{ij}(x)}{1-s_{ij}(x)}$.

4. Applications using Nevo (2000) and Berry, Levinsohn, and Pakes (1999)

To illustrate our theoretical results on diversion ratios, we use the two well-known examples from Nevo (2000b) and Berry et al. (1999). The data in Nevo (2000b) are simulated fake data meant to mimic the cereal industry from Nevo (2001), and consist of $T = 94$ markets, $J = 24$ products, and $N =$ 2,256 observations.[36] The Berry et al. (1999) data are annual national aggregate automobile sales by make and model for $T = 21$ markets (each market is a year),with up to $J = 150$ products per market and a total of $N =$ 2,217 observations. An advantage of the BLP data is that product names are attached, which help to interpret

[35] Many papers do not identify a heterogeneous price coefficient and thus α_i becomes a multiplicative constant in (15).

[36] These data are posted online by the author, and are not the actual data used in Nevo (2001), which are proprietary.

results.[37]

The specifications of utility in the two models are given by:

$$u_{ijt}^{\text{BLP}} = x_{jt} \underbrace{(\overline{\beta} + \Sigma \cdot \nu_i)}_{\beta_{it}} - \alpha \cdot \frac{p_{jt}}{\mathrm{y}_{it}} + \xi_{jt} + \varepsilon_{ijt}$$

$$u_{ijt}^{\text{Nevo}} = [-p_{jt}, x_{jt}] \underbrace{(\overline{\beta} + \Sigma \cdot \nu_i + \Pi \cdot y_{it})}_{\beta_{it}} + d_j + \Delta\xi_{jt} + \varepsilon_{ijt}$$

where ν_i denotes an unobserved characteristic of individual i and y_{it} denotes consumer demographics. The main difference between the two specifications is that Nevo (2000b) contains product fixed effects for each of the 24 brands d_j, and allows for interactions between household income and presence of children with tastes for product characteristics (the Π parameters). In the Berry et al. (1999) example, price is interacted with $1/y_{it}$ where y_{it} is a draw from a lognormal distribution of income.[38]

We estimate these parameters using the `PyBLP` software package of Conlon and Gortmaker (2020). In doing so, we implement the recommended "best practices," which involve calculating the nonlinear approximation to the optimal instruments. For the Nevo (2000b) example, we use the same 20 simulation draws provided in the original data package. For the Berry et al. (1999) example, the original draws are not available and we use 500 Halton draws. In addition to the baseline "best practices" estimates for each model, we consider three modified models in order to highlight how restrictions on parametric models are related to diversion ratios. In the first case, we eliminate all heterogeneous tastes for price $\pi_p = \sigma_p = 0$. In the second case we eliminate all heterogeneous tastes for the constant term $\sigma_0 = \pi_0 = 0$ (which governs "inside" vs. "outside" good substitution). We consider these two examples because many researchers consider these forms of heterogeneity to be among

[37] Product names are not available for the simulated Nevo (2000b) data.

[38] This is better behaved than $\alpha \cdot \log(y_i - p_{jt})$ from Berry et al. (1995). In estimation, Berry et al. (1999) also includes moment restrictions derived from an oligopolistic price-setting supply-side, which we include in estimation but do not report.

the most important to capture when modeling demand. In the final case, we rescale the market shares for all "inside" goods by a factor of three. This is meant to capture researcher uncertainty about the overall size of the market. In the Nevo (2000b) case we multiply $s_{jt} \cdot \frac{1}{3}$ and in the Berry et al. (1999) case we multiply $s_{jt} \cdot 3$.[39]

We report the parameter estimates in Section 5.[40] The top panel reports estimates for the Berry et al. (1999) example. Best practices estimates are reported in the first column. The second column reports estimates that omit heterogeneity on the constant term, which are similar. In the third column, we report estimates that omit heterogeneity on price, which differ significantly from the estimates in the first two columns. The last column rescales the market shares of all inside products, simulating a change in the assumed market size. As expected, the parameter estimates are sensitive to this change too. In the bottom panel, we perform the same comparisons for the Nevo (2001) example. In this case, we find that omitting heterogeneity on the price term gives estimates quite similar to 'best practices,' whereas omitting heterogeneity on the constant term and rescaling shares both lead to significantly different estimates.

For later comparisons, we also include a simple nested-logit model with all 'inside' products in a single nest. The nested-logit model offers a helpful comparison because, unlike the plain logit, it allows for a parameter that governs diversion to the outside good. Accordingly, the nested-logit is often used by researchers or antitrust agencies to provide a "first cut," or to model demand under time and/or data constraints. There are no random coefficients to report for the nested-logit model, so we do not include it in Table **??**. The only relevant parameter is the nesting parameter. In the Berry et al. (1999) example, we estimate the nesting parameter to be $\rho \approx 0.78$. For the Nevo (2001) example, we do not estimate a nesting

[39]In the BLP case this can be interpreted as an assumption that people look for a new car at most once every three years instead of every year. In the Nevo case this triples the potential number of servings (to breakfast, lunch, and dinner) that a household could consume each period.

[40]We only present the nonlinear parameters here. For the mean value of product characteristics consult Table B.1 in Appendix B.

parameter because the fake data have the same number of products in all markets, leaving us without the usual BLP-style instrument. Instead, we calibrate the nesting parameter to match the outside good diversion ($\rho = 0.375$).[41]

In Section 5 we explore the relationship between parametric restrictions on random-coefficients logit models and the resulting effects on elasticities and diversion ratios. In both the Nevo and BLP examples, the median own elasticity is relatively stable across random-coefficients specifications. The BLP and Nevo examples again struggle with different sets of restrictions, suggesting the models capture important sources of heterogeneity through different sets of parameters. Overall, the BLP example looks substantially different when we restrict heterogeneity on prices, and the Nevo example looks substantially different when we omit heterogeneous tastes for the constant, which governs inside vs. outside good substitution. Absent the key dimension of heterogeneity, diversion to the outside good increases from around 20% to around 89% in the BLP case, and from 33% to around 42% in the Nevo case.

We demonstrate a similar, though less interpretable, effect on the aggregate elasticity in both examples. The challenges with using aggregate elasticity as a diagnostic are highlighted when we rescale the shares. In the BLP example, we effectively triple the share of inside goods, which reduces outside-good diversion to around 16%; however, the aggregate elasticity doubles (from roughly 9% to 18%) because the outside-good share has declined. In the Nevo example, we effectively triple the share of the outside good, and thus outside-good diversion increases from around 33% to 78%, whereas the aggregate elasticity falls from about 53% to 38% (even though more consumers are diverted to the outside option). This suggests that outside-good diversion is a more useful diagnostic than the aggregate elasticity. In general, the lack of stability in the outside-good diversion (and aggregate elasticity) across

[41] This calibration is meant to represent a "best case" scenario for nested logit, or what we might hope to estimate if we had access to micro-moments or second-choice data.

specifications suggests this might be helped by additional (micro) moments or second-choice data.[42]

We also report (sales-weighted) substitution to the top-five substitutes for each product. The badly misspecified models (lacking price heterogeneity for BLP, or heterogeneity on the constant term for Nevo) understate substitution to the closest substitutes relative to the baseline model. Rescaling the inside shares for BLP leads to a slight increase in diversion to the best substitutes, and rescaling the outside share for Nevo leads to a substantial reduction in diversion to the closest substitutes, from about 40% to 13%. In both cases these effects appear to be related to the change in outside-good diversion. Markups (shown in (1) to be largely a function of own-elasticity and diversion to products of the same firm) appear to be more stable, but are still sensitive to outside-good diversion, with more outside-good diversion being associated with lower markups. As we showed in (15), consumer surplus is largely a function of three things: own share $s_{ij}(x)$, diversion to the outside good $D_{j0,i}(x)$, and price sensitivity α_i. Thus, it should be unsurprising that specifications that estimate different outside-good diversion also give wildly different consumer-surplus (CS) calculations (as does the BLP model without heterogeneous tastes for price).[43]

Although the nested-logit model matches the outside-good diversion reasonably well in both examples (by construction in the Nevo example), they do not capture the full extent of diversion to the top-five substitutes, because no parameters are left to explain similarities among pairs of products.[44] This also highlights that for these two examples, the "flexibility" of the random-coefficients specification is largely about capturing the inside/outside-good

[42] As an example, consider demand for distilled spirits as in Conlon and Rao (2019) or Miravete et al. (2018). We might also have quasi-experimental estimates (e.g., from a tax change on the total quantity sold within a particular state), and could use this as additional information.

[43] CS comparisons require care, because it is not possible to compare consumer utilities across specifications. However, these CS calculations are money-metric, in that $CS_i(x)$ is scaled by $\frac{1}{\alpha_i}$ for each individual, and then market averages are reported.

[44] Our estimation of the BLP nested logit omits the supply side, which leads to less elastic estimates of demand (and different markups and consumer surplus). Estimating the nested logit with all products in one nest and the same supply-side restrictions used by Berry et al. (1999) gives an estimate of $\rho \approx 1.4$.

diversion margin rather than similarities across pairs of products, which even for the closest substitutes are within 10% between the nested logit and baseline specifications.

Differences Among Treatment-Effects Estimates

As we discussed in Section 3, different "instruments" will recover different average measures of diversion ratios. An important question is how different these measures are from one another. For example, if the UK CMA were interested in the effect of a horizontal merger among retail supermarkets and surveyed potential customers asking about their second choice (e.g., "If this store were to close, where would you shop?"), one might want to know the difference between diversion estimated from a small price change $D_{jk}(p_j, x)$ and diversion estimated from the second-choice survey. As another example, an antitrust authority might want to know how consumers would respond to a reduction in the quality of cellular phone plans, but only had information from "number porting" data during a period in which consumer switching was driven largely by price cuts.[45]

In Section 3 and Table 2 we derived expressions for how these different interventions weighted different groups of individuals. Here we illustrate the implications of these different weights for the BLP and Nevo examples under the baseline specifications from the prior section. In our first exercise, we compare the diversion ratios obtained for every product in every market under the various "instruments" or interventions (small price changes, small quality changes, second-choice data, etc.).

In Table 5, we examine substitution to the best substitute for each product and to the outside good. For the BLP example, we measure diversion ratios for the best substitute for each vehicle, considering a small change in price or quality, or a product removal. All three random-coefficient measures are similar for the best substitute. We predict somewhat less

[45] A fully specified parametric demand model would answer these questions. However, in the first example, one may not be able to estimate a price parameter or elasticity, and in the second example preferences for "quality" might be subsumed into provider intercepts.

substitution to each product's closest substitute with the nested-logit specification. This is expected, because the nested-logit model lacks parameters that make a Camry similar to an Accord. The plain logit performs quite poorly, as it cannot properly capture diversion to the outside good, which in the case of the BLP data is extremely large.[46] Diversion to the outside good is larger when considering a small price change (around 20%) than when considering a change in quality or second-choice data (around 15%). This can be explained by differences in the weights in Table 2, and theoretically could go in either direction. The only difference between a marginal change in quality and second-choice data is a factor of $\frac{1}{1-s_{ij}(x)}$, but for most products $\frac{1}{1-s_{ij}(x)} \approx 1$ because the outside share is so large (often greater than 90%). However, responses to small changes in prices are substantially different because they place additional weight on more price-sensitive consumers (large $|\alpha_i|$). In the BLP example, price sensitivity is quite dispersed because it tracks income ($\alpha_i = \frac{\alpha}{\text{income}_i}$), and low-income consumers have much larger outside good shares $s_{i0}(x)$. We illustrate our decomposition of individual diversion measures and weights in more detail in Section 4.

We see a different pattern with the Nevo example. Here measures of diversion to the best substitute and diversion to the outside good are less sensitive to the particular weighting scheme that is used. This is despite the fact that there is still quite a bit of heterogeneity in the price sensitivity α_i as demonstrated by the magnitude of σ_{price} and $\pi_{\text{price}\times\text{inc}}$ and $\pi_{\text{price}\times\text{inc}^2}$ from Section 5.[47] This result is due to the fact that there is not as much heterogeneity in the underlying individual diversion ratios $D_{jk,i}(x)$ as demonstrated by two features of this market. First, the diversion ratios to each product's best substitute are similar between the nested-logit model, which predicts substitution proportional to share, and the random-coefficients model under any intervention. Second, beyond two discrete types of cereal (i.e., mushy and non-mushy), the extent of heterogeneous preferences for other characteristics in

[46] Berry et al. (1999) defines the market as every household in the United States purchasing up to one automobile each year.

[47] We provide histograms of α_i for both BLP and Nevo in Figure B.1 of the Appendix.

the Nevo application (i.e., sugar) is not very large.

This last point is highlighted by the "% Correct," which reports the fraction of observations for which each model predicts the same best substitute as the small-price change measure of diversion $D_{jk}(p_j, x)$. For the Nevo example, the three interventions identify the same best substitute at least 94% of the time. The nested-logit model does less well, identifying the top substitute only 64% of the time.[48] In the BLP example, nearly all interventions and specifications predict the same substitute for all products; thus, even the plain logit agrees with $D_{jk}(p_j, x)$ 95% of the time. What is perhaps disappointing for the BLP example is that diversion ratios to best substitutes are generally quite small (around 5%). This is in part driven by the larger number of products, but also suggests the possibility that even more random coefficients or demographic interactions might have been helpful.

In Table 6, we provide a standard econometric comparison among the different diversion measures. We use the small-price change case as our baseline, and compare (on the log-scale) the discrepancies between average diversion ratios measured using small quality changes, second-choice data, logit, and nested-logit models. We use two metrics for our comparison, the mean difference (bias), and the median absolute deviation (dispersion).[49] For the BLP example, we find that diversion to the top-five substitutes from alternative "instruments" (second-choice data and small-quality changes) are off by around 10% on average, with some being overstated and others being understated when compared to the small price change measure. As in Table 5 there are substantial discrepancies among the different "instruments" for the BLP data with second-choice and small-quality changes predicting around 30% less diversion to the outside good than a small price change, and around 27% more diversion to "all inside goods." The results are much more similar in the Nevo example, with the various random-coefficients diversion measures all within around 5% of the small-

[48] This is mostly because the nested-logit model cannot distinguish between mushy and non-mushy cereal.

[49] In Appendix B we provide a pairwise correlation matrix for these measures.

price-change predictions. As expected, the nested-logit model underpredicts substitution to the top substitutes and overpredicts substitution to "the field" of all inside goods.[50]

This raises the question: why do we see substantial discrepancies in diversion measured using small price changes versus small changes in quality or second-choice data in the BLP data, but not in the Nevo (fake) data?

Decomposition of Diversion Measures

In this section we demonstrate our decomposition from (7), which shows that we can decompose any measure of diversion ratios (second-choice data, small price changes, small quality changes) into two components: (1) an individual diversion measure $D_{jk,i}(x)$ that does not depend on the policy instrument; and (2) a set of weights $w_{ij}(x, z_j, z_j')$ that depend on the policy change, but do not depend on the identity of the substitute. The individual treatment effects (diversion ratios) are considered *structural parameters* in the sense that they are policy invariant (conditional on x) aspects of consumer preferences, which do not depend on whether we measure diversion by changing price or product quality (or by how much).

This policy invariance has led objects like $D_{jk,i}(x)$ to be described as *marginal-treatment effects* in the language of Heckman and Vytlacil (2005), as they can be integrated over different sets of weights to calculate the well-known treatment-effects parameters (ATE, ATT, ATUT, LATE, etc.).[51] To illustrate this connection, we present two sets of plots in Figure 1 and Figure 2. These are meant to parallel those in Carneiro et al. (2011), which plot the $MTE(x)$ against the propensity score (i.e., the probability that an individual is "treated").

In the first panel of each plot, we compute $D_{jk,i}(x) = \frac{s_{ik}(x)}{1-s_{ij}(x)}$ for each simulated "individual" in the Berry et al. (1999) model. We plot this against each individual's purchase probability for product j, s_{ij} because our treatment is "not buying j." Figure 1 plots diver-

[50] The nested logit does a good job matching the sales-weighted outside-good diversion in Table 5.

[51] This is demonstrated in (7).

sion measures from a Honda Accord to two substitutes: Toyota Camry and Cadillac DeVille. The plots reveal a large number of gray squares near the origin, representing individuals who are unlikely to purchase either an Accord or a Camry. Indeed, most consumers in the BLP data don't purchase any automobile at all. As we move right across the graph, we find that individuals who are more likely to purchase an Accord are also more likely to purchase a Camry. To help visualize this relationship, we provide a gray local linear smoothed line, which is increasing. This increasing relationship tells us about the covariance between the individual diversion measures and their weights in (6), and indicates that the products are close substitutes beyond what would be predicted by share alone. What is also interesting is that although these products are "close substitutes" even at the individual level, the highest diversion ratio we observe is around 8%.

As a contrasting case, in black in Figure 1 we conduct the same exercise but with the best-selling luxury car (the Cadillac DeVille) as the substitute instead. There are some individuals with diversion ratios as high as 10% between the Accord and the DeVille. However, because those individuals are on the left of the graph, these are individuals who are unlikely to buy the Accord in the first place (presumably because they are luxury car buyers, rather than economy car buyers). Although it is harder to see because of the large number of non-car buyers towards the bottom of the figure, the smoothed line slopes downwards, suggesting that individuals who really like the Accord are less likely to switch to the DeVille.

In the second panel of Figure 1 we plot a histogram for two different sets of weights across individuals. We plot the set of weights corresponding to a small price change $w_{ij}(x) = |\alpha_i| s_{ij}(x)(1 - s_{ij}(x))$, and the weights corresponding to second-choice data (or product removal) $w_{ij}(x) = s_{ij}(x)$. For the most part, the resulting histograms overlap, suggesting that the weights are largely similar for the two interventions. However, we see that second-choice weights tend to place slightly more weight on "more likely" Accord buyers (towards the right of the figure) than the small price change weights do.

The four horizontal dashed lines in the top panel of the figure indicate the average diversion from Accord to Camry (two dashed lines in grey) and from Accord to Cadillac DeVille (two dashed lines in black). The two measures for each pair of substitutes use the histograms in the lower panel to weight observations. The average diversion ratio from Accord to Camry obtained from a small price change is around 3.2%; measuring the same diversion ratio from second-choice data gives an estimate of average diversion of around 3.8%. The second-choice measure of average diversion is higher because the smoothed MTE for the Toyota Camry is upward sloping, and the histogram of weights for the second-choice measure places more weight on individuals to the right of the figure (with higher s_{ij}'s).

Even though there are some individuals with very high diversion ratios from the Accord to the DeVille, the average diversion ratios from a small price change or second-choice data are both quite small, $< 1\%$, because the high diversion individuals receive such low weight (presumably because they represent a tiny fraction of likely Accord buyers). We caution that the figures can still obfuscate some of the underlying heterogeneity. Two individuals may have similar s_{ij} "propensity scores" for the Accord, but for different reasons. The first may be on the margin of buying a car or not (low income, high price sensitivity). The second may be a buyer who is primarily shopping for luxury cars. Our figures may place these individuals at the same spot on the x-axis making this kind of multi-dimensional heterogeneity hard to visualize. This also explains why the relationship between the Accord and DeVille can be downward sloping but still have second-choice diversion that is higher than diversion from a small price change.

We observe a similar pattern in Figure 2, which considers diversion from the BMW 7-series to the BMW 5-series and the Mercedes 420s (three of the most expensive luxury vehicles in the dataset). Again, the majority of the points are around the origin, indicating that most simulated individuals don't purchase any of these high-end luxury vehicles. There is an increasing trend for diversion to both substitutes, as both the 5-series and the Mercedes are close substitutes to the 7-series. However, even among these close substitutes, the largest individual diversion ratio is less than 2.5%, and the average diversion measures are closer to 1.5% for the BMW 5-series (and 0.5% for the Mercedes).

Wald Estimates of Diversion Measures

A different way to decompose heterogeneity in the diversion ratio measures is to start with the same question we began this article with: what would we measure by considering a *ceteris paribus* increase in price from $p_j \to p'_j$? Different Wald estimators will produce different (local) average measures of diversion ratios. Consider the Wald estimate from (3):

$$\text{Wald}(z_j, z'_j, x) = \frac{q_k(z'_j, x) - q_k(z_j, x)}{\left|q_j(z'_j, x) - q_j(z_j, x)\right|}$$

For this example, we again focus solely on the BLP automobile data because there is less variation in the Nevo data in the underlying individual diversion measures $D_{jk,i}(x)$, and because the BLP data include product names that make the diversion ratios more interpretable.

We consider an increase in price $p_j \to p'_j$ or reduction in quality $\xi_j \to \xi'_j$ and plot the results for the Honda Accord and the BMW 7-series respectively in Figures 3 and 4, where the x-axis corresponds to the share of initial Accord (BMW) consumers who continue to purchase the Accord (BMW), and the y-axis corresponds to the Wald estimate of the diversion ratio. As we move toward the right, we consider larger price increases (or quality reductions) until the fraction of customers still purchasing the Accord or BMW approaches zero (the choke price). Dots on the graph denote 5% and 10% price increases, and a horizontal line marks the second-choice average (ATUT) diversion measure.

Figures 3 and 4 illustrate the fact that as we increase price (decrease quality) to the choke price (quality), the LATE estimates of diversion coincide with the second-choice (ATUT) measure. The figures also show that the second-choice data and Wald estimates using quality

(ξ_j) are more similar to one another than they are to the Wald estimates from price. This is likely to hold more generally, and is consistent with our findings in Table 5 because the weights from Table 2 for a second-choice measure of diversion, $\left(\widetilde{w}_{ij}^{\text{2nd choice}}(x) \propto \frac{s_{ij}(x)}{1-s_{ij}(x)}\right)$, and for a quality measure of diversion ($\widetilde{w}_{ij}^{\text{quality}}(x) \propto s_{ij}(x)$) are very similar, whereas the price change intervention places more weight on more price-sensitive individuals ($\widetilde{w}_{ij}^{\text{price}}(x) \propto |\alpha_i| s_{ij}(x)$).[52] In Figures 3 and 4, the average diversion ratio from a finite price change is smaller than the average diversion ratio obtained from quality changes or second-choice data. However, for other pairs of products, this pattern could be reversed. Indeed, the relationship between the LATE for a given finite price change, and a second-choice measure need not even be monotone (e.g., the measured (local) average diversion ratio may be increasing for small price changes, but decreasing for larger ones). More generally, the relationship between the average diversion for a finite price change vs. second-choice data is driven by the covariance between α_i and the corresponding shares (weights) $s_{ij}(x)$, as illustrated in the previous section. Without placing strong restrictions on the underlying heterogeneity, it is impossible to make statements like: "Small price changes lead to lower estimates of diversion ratios than second-choice data."[53]

Willingness-to-Pay (WTP) Measures

In Section 3, we demonstrate the relationship between the willingness-to-pay (WTP) measure (often used to analyze bargained outcomes and hospital mergers), the individual share $s_{ij}(x)$, the diversion to the outside good $D_{j0,i}(x)$, and the individual price sensitivity α_i. More specifically, in (15) we showed that $WTP_j = WTP_{ij} \partial F_i = \int D_{j0,i}(x) \frac{s_{ij}(x)}{|\alpha_i| \cdot s_{i0}(x)} \partial F_i = \int \frac{s_{ij}(x)}{|\alpha_i|(1-s_{ij}(x))} \partial F_i$. We can illustrate this relationship by considering the following two (de-

[52] This is particularly true if $\frac{s_{ij}(x)}{1-s_{ij}(x)} \approx s_{ij}(x)$ when shares are small. Here "price" is effectively any characteristic with a random coefficient and "quality" is the utility index or "vertical" characteristic. The LATE weights are proportional to $q(z_j, x) - q(z_j', x)$ rather than the derivative of $q_j(Z_j, x)$ at z_j.

[53] In some sense, this should be obvious because diversion ratios must sum to one, and for one diversion ratio to go up, others (including the outside good) must go down.

composition) regressions:

$$\begin{aligned} \ln WTP_{ijt} &= \beta_0 + \beta_1 \ln s_{jt,i}(x) + \beta_2 \ln D_{jt0,i}(x) + \beta_3 \ln |\alpha_{it}| & +\varepsilon_{ijt} \\ \ln WTP_{jt} &= \gamma_0 + \gamma_1 \ln s_{jt}(x) + \gamma_2 \ln D_{jt0}(x) & +\varepsilon_{jt} \end{aligned}$$

To run these regressions, we construct the WTP measure for every individual, in every market, for each product. For example, WTP_{ijt} might represent a particular household's willingness to pay to have a Honda Accord in their choice set in 1990; or it might represent their willingness to pay to have Apple Cinnamon Cheerios on the shelf of a supermarket in a particular week. WTP_{jt} represents the average of the individual measures across all households i.

We report the regression estimates in Table 7. At the individual level, we are able to explain over 99% of the variance in the individual-level WTP_{ij} measure using just $s_{jt,i}(x)$ and $D_{jt0,i}(x)$ for both the BLP and Nevo examples. In accordance with (15), the third and fourth columns demonstrate in the BLP application that once we control for the price sensitivity, $\ln(|\alpha_i|)$, diversion to the outside good at the individual level is no longer economically meaningful.

What is perhaps more surprising is that we are also able to explain average market-level WTP. The average WTP for access to a Camry or Cheerios in the choice set can be explained using only the *market-level* share $s_{jt}(x)$ and outside-good diversion $D_{j0}(x)$. In the Nevo (fake) data, we are able to explain 99.1% of the variance in WTP_j using just $s_{jt}(x)$ and 99.8% of variance once we also include the (market-level) diversion to the outside good. We see similar results for the BLP data, for which we explain roughly 78% of the variance using just $s_{jt}(x)$ and 91.3% of the variance in WTP_j using $s_{jt}(x)$ and market-level diversion to the outside good $D_{j0t}(x)$. We caution that at the aggregate level, this is neither a *sufficient statistic* representation nor a decomposition, in the sense that whereas the coefficient on

$\ln s_{jt}(x)$ is approximately one, the coefficient on $D_{j0t}(x)$ is not easily discerned without running the regression. In fact, it has a different sign in the Nevo data and the BLP data (because of how it covaries at the individual level with the price sensitivity $|\alpha_i|$).[54]

This highlights the fact that estimates of WTP are largely about own share $s_{jt}(x)$, which is often observed as data at the market level (and possibly at the individual level for some datasets – such as patient discharge data for hospitals). It also highlights the value of accurately measuring diversion to the outside good, including possibly from additional (micro) moments. Second-choice data, such as knowing what fraction of consumers would switch to a hospital outside the geographic market if a particular hospital were to close, may also be valuable such as in Raval et al. (2019).[55]

5. Conclusion

We provide a treatment-effects interpretation of diversion ratios, and demonstrate how different *ceteris paribus* interventions (price changes, quality changes, characteristic changes, and assortment changes) can be used to identify diversion ratios. Although these measures of diversion ratios will not in general agree with one another, we provide a framework to decompose these average measures into individual diversion ratios (which are policy-invariant structural parameters) and a set of weights (which vary with the intervention). We provide additional results showing how WTP is related to diversion to the outside good, and individual choice probabilities.

The object of interest may vary with the application. Sometimes the object of interest is a price change from a merger simulation or UPP calculation for a horizontal merger, in which case the researcher is likely interested in diversion ratios measured from small price

[54] Individual outside-good diversion is positively correlated with α_i in the BLP data and negatively correlated with α_i in the Nevo data. This may seem surprising at first, but the demographics (kids and income) and the relationship between price and the constant in the Nevo data induce the negative correlation.

[55] Recall that second-choice data integrates $s_{ik}(x)$ over the weights $\frac{s_{ij}(x)}{1-s_{ij}(x)}$, whereas WTP integrates $\frac{1}{|\alpha_i|}$ over the same set of weights, as demonstrated in Section 3.

changes. In other cases the relevant object may be the substitution induced from quality increases/reductions, or from changes in product assortment. In an ideal world, the object of interest would coincide with the data available to the researcher. In practice, researchers may have access to additional data from customer surveys, win-loss data, or customer switching behavior, which may or may not coincide with the diversion measure of interest. When these measures do not perfectly coincide, our framework provides a way to interpret discrepancies between different diversion measures.

In many cases researchers may still want to estimate a fully parametric model of demand (or supply and demand) such as in Berry et al. (1995) or Nevo (2000a). We hope that a better understanding of diversion ratios can help make the outputs of these models more interpretable. In simulations, we find that the diversion ratios measured from parametric models in two applications are perhaps more similar than one might have anticipated. For the automobile example, second-choice measures of diversion ratios are within 20% of measures based on small price changes, so that a 5% diversion ratio might be measured in the range of $(4\%, 6\%)$. For the simulated cereal data example, the estimated measures are even less dispersed, so that they are generally within 5% of one another. We caution that these findings are not guaranteed (or even likely) to hold in all potential examples, and that both settings have relatively large outside-good shares. This is important, because our simulations also indicate that measures of diversion ratios that rely on parametric models tend to be sensitive to the specification of the outside-good share or market size, indicating a potential need for additional data or moment restrictions (such as in Berry et al. (2004)) for this class of models.

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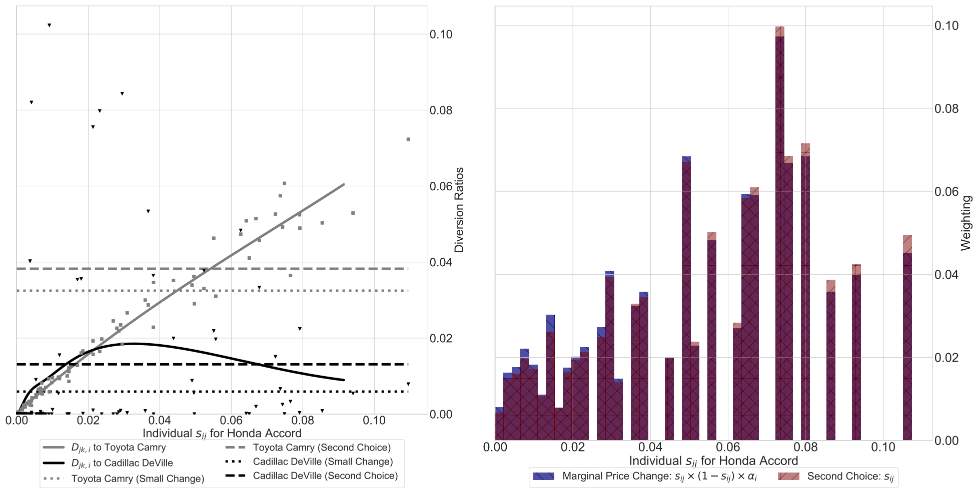
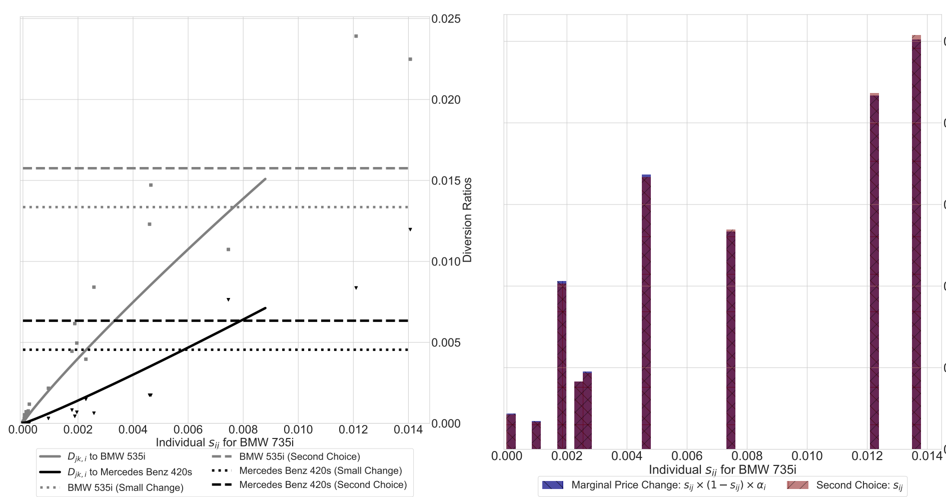
Figures and Tables

Figure 1: Decomposition of Diversion Measures for Honda Accord



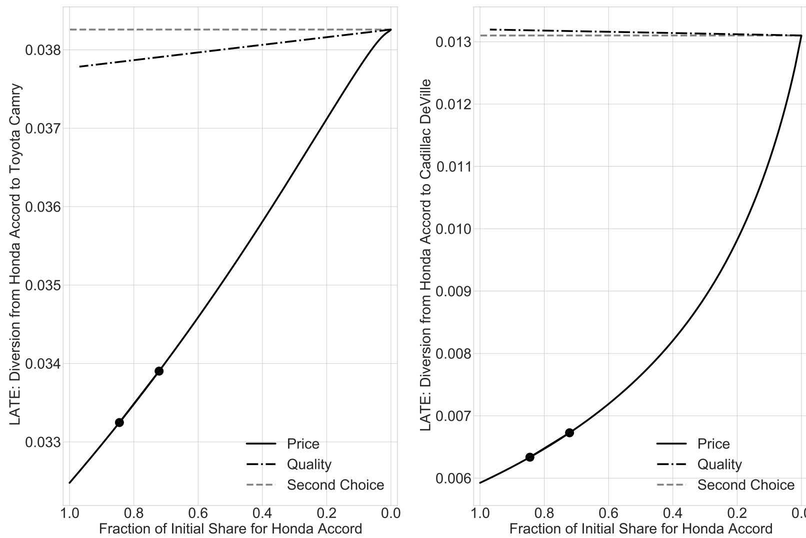
Notes: Figure shows diversion measures from a Honda Accord to two substitutes: Toyota Camry and Cadillac DeVille. The left panel plots $D_{jk,i}(x) = \frac{s_{ik}(x)}{1-s_{ij}(x)}$ for each simulated "individual" in the Berry et al. (1999) model against each individual's purchase probability for product j, s_{ij}. The right panel plots a histogram for two different sets of weights across individuals corresponding to: (i) a small price change $w_{ij}(x) = |\alpha_i| s_{ij}(x)(1 - s_{ij}(x))$, and (ii) second-choice data (or product removal) $w_{ij}(x) = s_{ij}(x)$. Sloped lines indicate the smoothed fitted relationship between s_{ij} and $D_{jk,i}(x)$ for each pair of substitutes. Horizontal lines indicate average diversion for each pair of substitutes under each set of weights.

Figure 2: Decomposition of Diversion Measures for BMW 7 Series



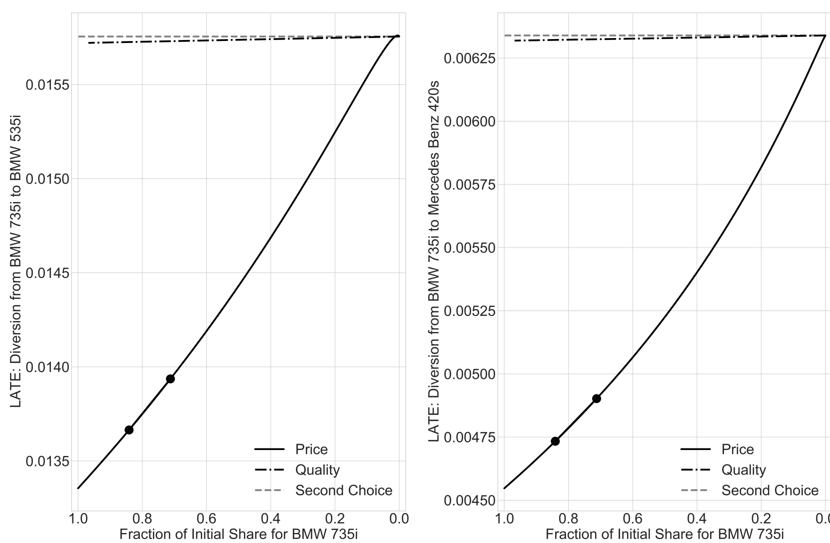
Notes: Figure shows diversion measures from a BMW 7 Series to two substitutes: BMW 535i and Mercedes Benz 420s. The left panel plots $D_{jk,i}(x) = \frac{s_{ik}(x)}{1-s_{ij}(x)}$ for each simulated "individual" in the Berry et al. (1999) model against each individual's purchase probability for product j, s_{ij}. The right panel plots a histogram for two different sets of weights across individuals corresponding to: (i) a small price change $w_{ij}(x) = |\alpha_i| s_{ij}(x)(1 - s_{ij}(x))$, and (ii) second-choice data (or product removal) $w_{ij}(x) = s_{ij}(x)$. Sloped lines indicate the smoothed fitted relationship between s_{ij} and $D_{jk,i}(x)$ for each pair of substitutes. Horizontal lines indicate average diversion for each pair of substitutes under each set of weights.

Figure 3: Local Average Diversion Measures from Accord to Camry and Deville



Notes: Figure shows average diversion calculated for different finite price and quality changes, and for second-choice (product removal) from Honda Accord to two substitutes: Toyota Camry and Cadillac DeVille. Marked points on the price curve denote LATEs for 5% and 10% price increases respectively.

Figure 4: Local Average Diversion Measures from BMW 7-series to 5-Series and Mercedes



Notes: Figure shows average diversion calculated for different finite price and quality changes, and for second-choice (product removal) from MBW 7 Series to two substitutes: BMW 535i and Mercedes Benz 420s. Marked points on the price curve denote LATEs for 5% and 10% price increases respectively.

Table 1: Description of Compliance Types and Treatment-Effects Parameters

Compliance Type	$(d_{ij}(p_j,x), d_{ij}(p'_j,x))$	Description
Always Takers	$(0,0)$	Don't buy j at either price.
Never Takers	$(1,1)$	Buy j at either price
Compliers	$(1,0)$	Only buy j at lower price $p_j < p'_j$
Defiers	$(0,1)$	Only buy j at higher prices $p'_j > p_j$
Treatment Effects Parameter	**Abbreviation**	**Expression**
Average Treatment Effect	ATE	$\mathbb{E}[D_{jk,i}(x)]$
Average Treatment on the Treated	ATT	$\mathbb{E}[D_{jk,i}(x)\|d_{ij}(p_j,x)=0]$
Average Treatment on the Untreated	ATUT	$\mathbb{E}[D_{jk,i}(x)\|d_{ij}(p_j,x)=1]$
Local Average Treatment Effect	LATE	$\mathbb{E}[D_{jk,i}(x)\|d_{ij}(p_j,x)=1, d_{ij}(p'_j,x)=0]$

Notes: Treatment is defined as "not buying j." Top panel defines compliance types; bottom panel reports definitions of treatment-effects parameters for diversion.

Table 2: Weighting of Different Treatment Effects Parameters under RC Logit

	$w_{ij}(x) \propto$	$\widetilde{w}_{ij}(x) \propto$
second choice data	$s_{ij}(x)$	$\frac{s_{ij}(x)}{1-s_{ij}(x)}$
price change $\frac{\partial}{\partial p_j}$	$s_{ij}(x)\cdot(1-s_{ij}(x))\cdot\|\alpha_i\|$	$s_{ij}(x)\cdot\|\alpha_i\|$
characteristic change $\frac{\partial}{\partial x_j}$	$s_{ij}(x)\cdot(1-s_{ij}(x))\cdot\|\beta_i\|$	$s_{ij}(x)\cdot\|\beta_i\|$
small quality change $\frac{\partial}{\partial \xi_j}$	$s_{ij}(x)\cdot(1-s_{ij}(x))$	$s_{ij}(x)$
finite price change $w_i(p_j,p'_j,x)$	$\|s_{ij}(p'_j,x)-s_{ij}(p_j,x)\|$	$\frac{\|s_{ij}(p'_j,x)-s_{ij}(p_j,x)\|}{1-s_{ij}(x)}$
finite quality change $w_i(\xi_j,\xi'_j,x)$	$\|s_{ij}(\xi'_j,x)-s_{ij}(\xi_j,x)\|$	$\frac{\|s_{ij}(\xi'_j,x)-s_{ij}(\xi_j,x)\|}{1-s_{ij}(x)}$
willingness to pay (WTP)	$=\frac{s_{ij}(x)}{\|\alpha_i\|\cdot s_{i0}(x)}$	$\frac{s_{ij}(x)}{\|\alpha_i\|\cdot s_{i0}(x)(1-s_{ij}(x))}$

Notes: Weights are used to construct the weighted average diversion measure: $\sum_i D_{jk,i}(x)w_{ij}(x)$. Weights must be normalized to integrate to one $\int w_{ij}(x)dF_i = 1$, but note that $\int \widetilde{w}_{ij}(x)dF_i \neq 1$. The weights $w_{ij}(x)$ for the WTP measure are as written (do not integrate to one).

Table 3: Parameter Estimates for Berry et al. (1999) and Nevo (2000b).

	Best Practices	$\Sigma_{\text{cons}} = \pi_{\text{cons}} = 0$	$\Sigma_p = \pi_p = 0$	Rescaled Shares
BLP				
price/inc	-51.254	-49.175	-0.355	-11.277
	(5.847)	(7.104)	(0.030)	(2.218)
σ_{cons}	2.052	–	0.024	3.159
	(1.111)	–	(35.915)	(1.364)
$\sigma_{\text{HP/weight}}$	1.785	2.661	0.097	1.257
	(2.061)	(1.258)	(33.495)	(2.257)
σ_{air}	1.899	1.135	0.080	1.408
	(0.439)	(0.522)	(133.351)	(0.445)
$\sigma_{\text{MP\$}}$	0.708	0.157	0.003	0.151
	(0.184)	(0.223)	(14.096)	(0.361)
σ_{size}	1.126	1.554	0.012	0.261
	(0.917)	(0.736)	(29.555)	(2.405)
Nevo				
α_{price}	-31.125	-42.642	-30.939	-36.280
	(4.700)	(3.627)	(0.913)	(2.507)
σ_{price}	2.983	1.362	–	2.314
	(0.650)	(0.741)	–	(0.537)
σ_{cons}	0.217	–	0.246	0.016
	(0.078)	–	(0.079)	(0.070)
σ_{sugar}	0.027	0.055	0.055	0.028
	(0.007)	(0.012)	(0.012)	(0.007)
σ_{mushy}	0.294	0.720	0.835	0.172
	(0.101)	(0.339)	(0.302)	(0.089)
$\pi_{\text{price}\times\text{inc}}$	92.746	49.645	–	214.812
	(89.324)	(66.938)	–	(40.954)
$\pi_{\text{price}\times\text{inc}^2}$	-5.266	-1.535	–	-11.596
	(4.621)	(3.492)	–	(2.143)
$\pi_{\text{price}\times\text{kids}}$	4.056	0.186	–	3.537
	(2.260)	(2.511)	–	(2.021)
$\pi_{\text{cons}\times\text{inc}}$	6.043	–	4.819	4.071
	(0.538)	–	(0.357)	(0.435)
$\pi_{\text{cons}\times\text{age}}$	0.161	–	-0.049	0.018
	(0.203)	–	(0.200)	(0.194)
$\pi_{\text{sugar}\times\text{inc}}$	-0.310	-0.088	-0.245	-0.287
	(0.035)	(0.030)	(0.023)	(0.035)
$\pi_{\text{sugar}\times\text{age}}$	0.049	0.023	0.036	0.046
	(0.013)	(0.016)	(0.016)	(0.015)
$\pi_{\text{mushy}\times\text{inc}}$	0.982	1.193	0.741	0.805
	(0.279)	(0.310)	(0.261)	(0.266)
$\pi_{\text{mushy}\times\text{age}}$	-0.537	-0.029	-0.176	-0.508
	(0.181)	(0.237)	(0.230)	(0.180)

Notes: *Best practices* refers to estimates with optimal instruments. Conlon and Gortmaker (2020) provides details. The BLP example uses 500 Halton draws to approximate the numerical integral and includes supply side. The Nevo (2000b) example uses the same 20-point distribution of heterogeneity included with the (simulated) data. *Rescaled Shares* uses $s_j \times 3$ for BLP and $s_j \times \frac{1}{3}$ for Nevo.

Table 4: Comparison of Diversion and Elasticities for Berry et al. (1999) and Nevo (2000b).

	Best Practices	$\Sigma_{\text{cons}} = \pi_{\text{cons}} = 0$	$\Sigma_p = \pi_p = 0$	Rescaled Shares	Nested Logit
BLP					
Median Own-Elasticity	-3.811	-3.481	-3.098	-2.632	-1.600
Median Aggregate Elasticity	-0.096	-0.095	-0.258	-0.178	-0.033
Median Outside-Good Diversion	0.201	0.227	0.892	0.163	0.197
Mean Top 5 Diversion	0.182	0.165	0.022	0.191	0.165
Mean Markup	0.334	0.359	0.385	0.468	0.936
Median Consumer Surplus	2.071	2.005	0.323	14.726	2.827
Nevo					
Median Own-Elasticity	-3.654	-3.735	-3.686	-3.622	-3.995
Median Aggregate Elasticity	-0.534	-0.683	-0.558	-0.383	-0.584
Median Outside-Good Diversion	0.329	0.418	0.341	0.780	0.342
Mean Top 5 Diversion	0.406	0.334	0.392	0.129	0.385
Mean Markup	0.402	0.384	0.402	0.324	0.363
Median Consumer Surplus	2.946	3.750	3.218	0.628	3.011

Notes: Mean markup and diversion are sales-weighted.
Nested Logit (BLP): all products in single nest and estimates $\rho = 0.78$.
Nested Logit (Nevo): all products in single nest and calibrates $\rho = 0.375$ to match outside good diversion.

Table 5: Diversion to Best Substitute and Outside Good

	$D_{jk}(p)$	Small Quality Change	Second Choice	Logit	Nested Logit $D_{jk}(p)$
BLP					
Med(D_{jk})	4.53	4.62	4.63	0.46	3.85
Mean(D_{jk})	5.11	5.30	5.33	0.53	3.97
% Correct	100.00	97.29	97.29	95.58	95.58
Med(D_{j0})	20.08	14.40	14.33	89.26	19.66
Mean(D_{j0})	20.29	14.95	14.88	89.36	19.71
Nevo					
Med(D_{jk})	13.03	12.83	12.90	8.89	12.54
Mean(D_{jk})	15.29	14.79	14.90	9.84	13.82
% Correct	100.00	94.33	94.46	64.23	64.23
Med(D_{j0})	32.91	34.13	34.00	54.43	34.20
Mean(D_{j0})	32.88	34.08	33.85	53.46	33.73

Notes: The first panel in each set reports diversion to each product-market pair's best substitute. The second panel in each set reports diversion to the outside good.

Table 6: Relative % Difference in Diversion Measures

	BLP		Nevo	
	med($\lvert y-x\rvert$)	mean($y-x$)	med($\lvert y-x\rvert$)	mean($y-x$)
Top Five Substitutes				
Small Quality Change	10.57	2.49	3.66	-2.73
Second Choice	10.47	2.81	3.34	-2.03
Logit	19.53	-22.38	38.80	-44.41
Nested Logit $D_{jk}(p)$	19.06	-6.48	19.06	-6.48
All Products				
Small Quality Change	21.25	27.94	4.92	-0.55
Second Choice	21.39	27.85	4.63	-0.64
Logit	191.18	-170.71	38.64	-29.47
Nested Logit $D_{jk}(p)$	33.71	32.73	32.22	8.46
Outside Good				
Small Quality Change	32.62	-31.63	3.86	5.40
Second Choice	33.03	-32.19	3.25	4.35
Logit	149.38	168.63	44.63	58.46
Nested Logit $D_{jk}(p)$	39.28	17.46	29.95	12.33

Notes: Baseline is small-price change measure: $D_{jk}(p_j)$.
Observations are product-market pairs and are *equally* weighted.
Median Absolute Deviation med($|y-x|$) and "Bias" $E[y-x]$ where (x,y) are log-diversion.

Table 7: Correlation with WTP Measure for Nevo (2000b) and Berry et al. (1999).

	Individual						Aggregate			
	BLP	BLP	BLP	BLP	Nevo	Nevo	BLP	BLP	Nevo	Nevo
$\log(s_{jt})$	1.0749*	1.0519*	0.9955*	0.9955*	1.0255*	1.0353*	0.7669*	0.9265*	1.0144*	1.0105*
	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0002)	(0.0002)	(0.0087)	(0.0061)	(0.0020)	(0.0010)
$\log(D_{j,0})$		-0.2233*	-0.0032*			0.1090*		-0.6733*		0.1786*
		(0.0005)	(0.0002)			(0.0009)		(0.0114)		(0.0022)
$\log(\lVert\alpha_i\rVert)$			-1.0113*	-1.0164*						
			(0.0005)	(0.0004)						
R^2	0.9951	0.9967	0.9997	0.9997	0.9975	0.9981	0.7784	0.9139	0.9915	0.9978
Adjusted R^2	0.9951	0.9967	0.9997	0.9997	0.9975	0.9981	0.7783	0.9138	0.9915	0.9978
Observations	436199	436199	436199	436199	45051	45051	2217	2217	2256	2256

Notes: For individual-draw specifications $s_{ijt}(x)$ and $D_{j0,i}(x)$ are used. Approximately half of all observations for Berry et al. (1999) $WTP(i,j)$ are excluded because $WTP(i,j) \approx 0$ to machine precision.

Appendices

A. Theory Appendix

A.1. Proofs and Derivations

Analogue to LATE Theorem (Imbens and Angrist, 1994)]

Under the following conditions:

(a) Mutually Exclusive and Exhaustive Discrete Choice: $d_{ij} \in \{0,1\}$ *and* $\sum_{j\in\mathcal{J}} d_{ij} = 1$.

(b) Exclusion: $u_{ik}(p_j) = u_{ik}(p'_j)$ *for all* $k \neq j$ *and any* (p_j, p'_j)*;*

(c) Monotonicity: $u_{ij}(p'_j) \leq u_{ij}(p_j)$ *for all* i *and any* $(p'_j > p_j)$*; and*

(d) Existence of a first-stage: $d_{ij}(p_j, x) = 1$ *and* $d_{ij}(p'_j, x) = 0$ *for* $(p'_j > p_j)$ *for some* i*;*

(e) Random Assignment: $(u_{ij}(P_j), u_{ik}(P_j)) \perp P_j$,

then the Wald estimator from (3)*:*

$$\frac{q_k(p'_j, x) - q_k(p_j, x)}{-\big(q_j(p'_j, x) - q_j(p_j, x)\big)} = \mathbb{E}[D_{jk,i}(x) | d_{ij}(p_j, x) > d_{ij}(p'_j, x)].$$

Proof of Proposition 1:

We suppress x because everything is done conditional on x. We begin by observing under the discrete choice assumption $q_k = M \cdot s_k$ where s_k is the market share of good k (including an outside option) and M is the size of the market:

$$\frac{q_k(p'_j, x) - q_k(p_j, x)}{-\big(q_j(p'_j, x) - q_j(p_j, x)\big)} = \frac{s_k(p'_j, x) - s_k(p_j, x)}{-\big(s_j(p'_j, x) - s_j(p_j, x)\big)} \tag{A.1}$$

We also use the definition that $s_k(P_j, x) = \mathbb{E}[d_{ik}(P_j) = 1|x] = \mathbb{E}[d_{ik}(P_j)|x]$. This says the market share is equal to the average purchase probability when choices are discrete, exhaustive, and mutually exclusive and follows from (a). In all cases, the expectation operator $\mathbb{E}(\cdot)$ averages over the distribution of (heterogeneous) individuals i in the population F_i.

Define $\overline{u}_i = \max_{k'} u_{ik'}$ for $k' \in \mathcal{J} \setminus \{j,k\}$ (the best product for i other than (j,k)). Notice that $\overline{u}_i(p_j) = \overline{u}_i(p'_j)$ for any (p_j, p'_j) (the Exclusion restriction). Consider the numerator:

$$\begin{aligned}
\mathbb{E}[d_{ik}(P_j)|P_j = p'_j] &= Pr(u_{ik}(p'_j) > u_{ij}(p'_j)\,;\, u_{ik}(p'_j) > \overline{u}_i(p'_j)|P_j = p'_j) \\
&= Pr(u_{ik} > u_{ij}(p'_j)\,;\, u_{ik} > \overline{u}_i|P_j = p'_j) \quad \text{by exclusion} \\
&= Pr(u_{ik} > u_{ij}(p'_j)\,;\, u_{ik} > \overline{u}_i) \quad \text{by random assignment}
\end{aligned}$$

This means we can re-write the numerator:

$$\begin{aligned}
\mathbb{E}[d_{ik}(P_j)|P_j = p'_j] - \mathbb{E}[d_{ik}(P_j)|P_j = p_j] &= Pr(u_{ik} > u_{ij}(p'_j)\,;\, u_{ik} > \overline{u}_i) - Pr(u_{ik} > u_{ij}(p_j)\,;\, u_{ik} > \overline{u}_i) \\
&= Pr(u_{ij}(p'_j) < u_{ik} < u_{ij}(p_j)\,;\, u_{ik} > \overline{u}_i) \quad \text{by Monotoncity} \\
&= Pr(u_{ik} > u_{ij}(p'_j)\,;\, u_{ij}(p_j) > u_{ik}\,;\, u_{ik} > \overline{u}_i) \\
&= Pr(\max\{u_{ik}, \overline{u}_i\} > u_{ij}(p'_j)\,;\, u_{ij}(p_j) > \max\{u_{ik}, \overline{u}_i\}\,;\, u_{ik} > \overline{u}_i) \\
&= Pr\left(u_{ik} > \overline{u}_i\,;\, d_{ij}(p'_j) = 0\,;\, d_{ij}(p_j) = 1\right) \quad \text{by defn } d_{ij}
\end{aligned}$$

The denominator, beginning with the exclusion restriction:

$$\begin{aligned}
E[d_{ij}(P_j)|P_j = p'_j] &= 1 - Pr(u_{ij}(p'_j) > u_{ik}(p'_j)\,;\, u_{ij}(p'_j) > \overline{u}_i(p'_j)|P_j = p'_j) \\
&= 1 - Pr\left(u_{ij}(p'_j) > \max\{u_{ik}, \overline{u}_i\}\right) \quad \text{by random assignment}
\end{aligned}$$

And we can re-write the denominator:

$$\begin{aligned}
E[d_{ij}(P_j)|P_j = p'_j] - E[d_{ij}(P_j)|P_j = p_j] &= Pr\left(u_{ij}(p_j) > \max\{u_{ik}, \overline{u}_i\}\right) - Pr\left(u_{ij}(p'_j) > \max\{u_{ik}, \overline{u}_i\}\right) \\
&= Pr\left(u_{ij}(p'_j) < \max\{u_{ik}, \overline{u}_i\} < u_{ij}(p_j)\right) \quad \text{by Monotoncity} \\
&= Pr\left(\max\{u_{ik}, \overline{u}_i\} > u_{ij}(p'_j)\,;\, u_{ij}(p_j) > \max\{u_{ik}, \overline{u}_i\}\right) \\
&= Pr\left(d_{ij}(p'_j) = 0\,;\, d_{ij}(p_j) = 1\right) \quad \text{by defn } d_{ij}
\end{aligned}$$

The ratio in (A.1) is:[56]

$$
\begin{aligned}
\frac{E[d_{ik}(p'_j,x)]-E[d_{ik}(p_j,x)]}{E[d_{ij}(p'_j,x)]-E[d_{ij}(p_j,x)]} &= \frac{Pr\left(u_{ik}>\overline{u}_i\,;\,d_{ij}(p'_j)=0\,;\,d_{ij}(p_j)=1\right)}{Pr\left(d_{ij}(p'_j)=0\,;\,d_{ij}(p_j)=1\right)}\\
&= Pr\left(u_{ik}>\overline{u}_i\,|\,d_{ij}(p'_j)=0\,;\,d_{ij}(p_j)=1\right) \qquad \text{by Bayes' Rule and (d)}\\
&= E\left[\mathbf{1}[u_{ik}(x)>\overline{u}_i(x)]\,|\,d_{ij}(p'_j,x)<d_{ij}(p_j,x)\right]\\
&= E\left[D_{jk,i}(x)\,|\,d_{ij}(p'_j,x)<d_{ij}(p_j,x)\right]\Box.
\end{aligned}
$$

The last line is merely the definition of the individual diversion ratio: that individual i prefers k to any non j alternative so that i's "treatment effect" is given by: $Y_i(1)-Y_i(0)=\mathbf{1}[u_{ik}>\overline{u}_i]$. Notice this satisfies the "Independence" assumption of Imbens and Angrist (1994). That is, $D_{jk,i}(x)=\mathbf{1}[u_{ik}(x)>\overline{u}_i(x)]$ does not depend on the "instrument" P_j.

In words, the Wald estimator delivers the expected treatment effect on the compliers (i.e.: individuals for whom $(d_{ij}(p_j)=1, d_{ij}(p'_j)=0)$. The compliers are the individuals who would have purchased j at price p_j and who do not purchase j at price p'_j.

A.2. Diversion Under Common Parametric Demands

This section derives explicit formulas for the diversion ratio under common parametric forms for demand, focusing on whether or not the diversion ratio implied by a particular parametric form of demand is constant with respect to the magnitude of the price increase. We show that the plain 'IIA' logit and linear demand models exhibit this property, while the log-linear and mixed logit models do not necessarily exhibit this property. We go through several derivations below.

[56] This requires the "first stage" assumption to avoid division by zero: $Pr\left(d_{ij}(p'_j)=0\,;\,d_{ij}(p_j)=1\right)\neq 0$.

Linear Demand

The diversion ratio under linear demand has the property that it does not depend on the magnitude of the price increase. We specify linear demand as:

$$Q_k(P_1, \ldots, P_J) = \alpha_k + \sum_j \beta_{kj} P_j.$$

where $\frac{\partial Q_k}{\partial P_j} \equiv \beta_{kj}$ is the increase or decrease in k's quantity due to a one-unit increase in proupt j's price.

$$D_{jk}(P_1, \ldots, P_J, x) = \frac{\beta_{kj}}{\beta_{jj}} \tag{A.2}$$

Thus, for any change in p_j from an infinitesimal price increase up to the choke price of j, the diversion ratio D_{jk} is constant. This also implies that under linear demand, diversion is a global property. Any magnitude of price increase evaluated at any initial set of prices and quantities will result in the same measure of diversion.

Log-Linear Demand

The log-linear demand model does not exhibit constant diversion with respect to the magnitude of the price increase. The log-linear model is specified as:

$$\ln(Q_k) = \alpha_k + \sum_{\ell} \epsilon_{k\ell} \ln(P_j)$$

As is well known $\epsilon_{jk} = \frac{\partial Q_k}{\partial P_j} \frac{P_j}{Q_k}$ and the diversion ratio can be re-written as:

$$D_{jk}(P_j, x) = \frac{Q_k(P_j, x)}{Q_j(P_j, x)} \cdot \frac{\epsilon_{kj}}{\epsilon_{jj}}$$

Under constant elasticity, the diversion ratio depends on where on the demand curve P_j is located. (This is similar to the fact that under linear demand, the own-price elasticity depends on where one analyzes the demand curve.) As an additional challenge, second-choice data are not interpretable because we cannot expect to set $Q_j = 0$ and take $\log(Q_j)$.

Plain 'IIA' Logit Demand

The plain logit model exhibits Independence of Irrelevant Alternatives (IIA) and proportional substitution, which implies that the diversion ratio does not depend on the magnitude of the price increase. The logit utility and market shares $q_j(P_j, x) = s_j(P_j, x) \cdot M$ are given by the well-known formulas:

$$\begin{aligned} u_{ij}(P_j, x) &= \beta x_j - \alpha P_j + \xi_j + \varepsilon_{ij} \\ s_j(P_1, \ldots, P_j, x) &= \frac{\exp[\beta x_j - \alpha P_j + \xi_j]}{1 + \sum_{k \in \mathcal{J}} \exp[\beta x_k - \alpha P_k + \xi_k]} \end{aligned}$$

The logit derivatives also have well-known formulas:

$$\begin{aligned} \frac{\partial s_j}{\partial P_j} &= \alpha s_j(1 - s_j) & \frac{\partial s_k}{\partial P_j} &= -\alpha s_j s_k \\ \frac{\partial^2 s_j}{\partial P_j^2} &= \alpha^2(1 - 2s_j)(s_j - s_j^2) & \frac{\partial^2 s_k}{\partial P_j^2} &= -\alpha^2(1 - 2s_j)s_j s_k \end{aligned}$$

This means that under a logit demand the diversion from an infinitesimal price change is given by:

$$\frac{\frac{\partial s_k}{\partial P_j}}{\left|\frac{\partial s_j}{\partial P_j}\right|} = \frac{\alpha s_k s_j}{\alpha s_j(1 - s_j)} = \frac{s_k}{(1 - s_j)} \tag{A.3}$$

Meanwhile the diversion ratio exhibits constant treatment effects IFF the numerator in (10) is zero. This is true if $D_{jk}(P_j,x) = -\frac{\frac{\partial^2 q_k}{\partial P_j^2}}{\frac{\partial^2 q_j}{\partial P_j^2}}$. This property is easily verified for the logit:

$$-\frac{\frac{\partial^2 q_k}{\partial P_j^2}}{\frac{\partial^2 q_j}{\partial P_j^2}} = \frac{\alpha^2(1-2s_j)s_j s_k}{\alpha^2(1-2s_j)(s_j - s_j^2)} = \frac{s_k}{1-s_j}.$$

For second-choice data, it is helpful to define exponentiated indirect utility: $V_k = \exp[\beta x_j - \alpha P_j + \xi_j]$ and write:

$$\begin{aligned}
\frac{s_k(\mathcal{J},x) - s_k(\mathcal{J}\setminus j,x)}{s_j(\mathcal{J},x)} &= \frac{\frac{V_k}{1+\sum_{j'\in\mathcal{J}\setminus j} V_{j'}} - \frac{V_k}{1+\sum_{j'\in\mathcal{J}} V_{j'}}}{\frac{V_j}{1+\sum_{j'\in\mathcal{J}} V_{j'}}} \\
&= \frac{V_k}{1+\sum_{j'\in\mathcal{J}} V_{j'}} \cdot \left(\frac{1+\sum_{j'\in\mathcal{J}} V_{j'}}{1+\sum_{j'\in\mathcal{J}\setminus j} V_{j'}} - 1\right)\frac{1}{V_j} = \frac{s_k(\mathcal{J},x)}{1-s_j(\mathcal{J},x)}
\end{aligned}$$

In both cases, diversion is the ratio of the change in the market share of the substitute good divided by the share of consumers no longer buying the focal good (under the initial set of prices and product availability). It does not depend on any of the estimated parameters (α,β). It is also true that all individual diversion ratios are equal $D_{jk,i}(x) = \frac{s_k(x)}{1-s_j(x)}$. This is also equal to the ATT for any discrete-choice model: $ATT = E[D_{jk,i}(x)|d_{ij}(x)=0]$. Thus, plain 'IIA' logit implies no selection into treatment.

Nested-Logit Demand

Recall the estimating equation for the nested logit from Berry (1994):

$$\ln s_{jt} - \ln s_{0t} = x_{jt}\beta - \alpha p_{jt} + \sigma \ln s_{j|g,t} + \varepsilon_{jt}$$

The derivatives of market share with respect to price are given by:[57]

$$
\begin{aligned}
\frac{\partial s_j}{\partial P_j} &= \alpha s_j \left(\frac{-1}{1-\sigma} + \frac{\sigma}{1-\sigma} s_{j|g} + s_{jt} \right) \\
\frac{\partial s_k}{\partial P_j} &= \left\{ \begin{array}{ll} \alpha s_j \left(\frac{\sigma}{1-\sigma} s_{k|g} + s_k \right) & \text{for } (j,k) \text{ in same nest} \\ \alpha s_j s_k & \text{otherwise} \end{array} \right\}
\end{aligned}
$$

And second derivatives:

$$
\begin{aligned}
\frac{\partial^2 s_j}{\partial^2 P_j} &= \alpha^2 \left(\frac{1}{1-\sigma} - \frac{\sigma}{1-\sigma} \bar{s}_{j|g} - 2 s_j \right) \left(\frac{1}{1-\sigma} - \frac{\sigma}{1-\sigma} \bar{s}_{j|g} - s_j \right) s_j \\
&\quad - \alpha^2 \frac{\sigma}{(1-\sigma)^2} s_j \bar{s}_{j|g} \left(1 - \bar{s}_{j|g} \right) \\
\frac{\partial^2 s_k}{\partial^2 P_j} &= \alpha^2 s_k s_j \left(s_j - \frac{1}{1-\sigma} \left(1 - \sigma \bar{s}_{j|g} - (1-\sigma) s_j \right) \right) \quad \text{for different groups .} \\
\frac{\partial^2 s_k}{\partial^2 P_j} &= -\alpha^2 \frac{1}{1-\sigma} s_k \left(1 - \sigma \bar{s}_{k|g} - (1-\sigma) s_k \right) \left(s_j + \frac{\sigma}{1-\sigma} \bar{s}_{j|g} \right) \\
&\quad + \alpha^2 s_k^2 \left(s_j + \frac{\sigma}{1-\sigma} \bar{s}_{j|g} \right) + \alpha^2 \frac{\sigma}{1-\sigma} \frac{1}{1-\sigma} s_k \bar{s}_{j|g} \bar{s}_{k|g} \quad \text{for same groups .}
\end{aligned}
$$

It is helpful to define $Z(\sigma, s_g) = [\sigma + (1-\sigma)s_g] \in (0,1]$ and note that $Z(0, s_g) = s_g$ and $Z(1, s_g) = 1$. When both products are in the same nest, the diversion ratio is given by:

$$
\begin{aligned}
-\frac{\frac{\partial s_k}{\partial P_j}}{\frac{\partial s_j}{\partial P_j}} &= -\frac{\frac{\sigma}{1-\sigma} s_{k|g} + s_k}{\frac{-1}{1-\sigma} + \frac{\sigma}{1-\sigma} s_{j|g} + s_j} = -\frac{\sigma s_{k|g} + s_{k|g} s_g (1-\sigma)}{-1 + \sigma s_{j|g} + (1-\sigma) s_{j|g} s_g} = -\frac{s_{k|g}[\sigma + (1-\sigma)s_g]}{-1 + s_{j|g}[\sigma + (1-\sigma)s_g]} \\
&= \frac{s_{k|g} \cdot Z(\sigma, s_g)}{1 - s_{j|g} \cdot Z(\sigma, s_g)} = \frac{s_{k|g}}{Z^{-1}(\sigma, s_g) - s_{j|g}} \equiv D^*_{jk} \qquad \text{(A.4)}
\end{aligned}
$$

Observe that $Z^{-1}(\sigma, s_g) \geq 1$. Also notice that diversion ratios are proportional to within-group share $s_{k|g}$.

If two products are in different nests, we need to re-scale the numerator by $\frac{s_k}{\frac{\sigma}{1-\sigma} s_{k|g} + s_k} = \frac{s_g(1-\sigma)}{\sigma + (1-\sigma)s_g} = s_g(1-\sigma)Z^{-1}(\sigma, s_g)$. We must also keep track of which product's group share we

[57] See Mansley et al. (2019) for derivatives worked out.

are referring to. It is helpful to use the second-to-last expression in the derivation for the same nest:

$$-\frac{\frac{\partial s_k}{\partial P_j}}{\frac{\partial s_j}{\partial P_j}} = \frac{s_{k|g} \cdot Z(\sigma, s_{g(k)})}{1 - s_{j|g} \cdot Z(\sigma, s_{g(j)})} \cdot s_{g(k)}(1-\sigma) Z^{-1}(\sigma, s_{g(k)})$$
$$= \frac{s_k(1-\sigma)}{1 - s_{j|g} \cdot Z(\sigma, s_{g(j)})} \equiv D^{**}_{jk} \tag{A.5}$$

Notice that we get diversion ratios proportional to overall share s_k.

We can relate the diversion ratio of two products within the group D^*_{jk} to the diversion ratio of two products in different groups D^{**}_{jk} by:

$$D^{**}_{jk} = D^*_{jk} \cdot \frac{s_{g(k)} \cdot (1-\sigma)}{Z(\sigma, s_{g(j)})}$$

As $\sigma \to 1$ everyone stays within the group and $D^*_{jk} = \frac{s_{k|g}}{1-s_{j|g}}$ and $D^{**}_{jk} = 0$.
As $\sigma \to 0$ we collapse to the logit and $D^*_{jk} = D^{**}_{jk} = \frac{s_{k|g}}{\frac{1}{s_g} - s_{j|g}} \cdot \frac{s_g}{s_g} = \frac{s_k}{1-s_j}$.

We can also check the curvature property in (9). We are interested in the ratio of $\frac{\partial^2 s_k}{\partial^2 P_j}$ to $\frac{\partial^2 s_j}{\partial^2 P_j}$, which for two products in the same group, is given by:

$$-\frac{\frac{\partial^2 s_k}{\partial^2 P_j}}{\frac{\partial^2 s_j}{\partial^2 P_j}} = -\frac{s_k s_j \left(s_j - \frac{1}{1-\sigma}\left(1 - \sigma \bar{s}_{j|g} - (1-\sigma)s_j\right)\right)}{\left(\frac{1}{1-\sigma} - \frac{\sigma}{1-\sigma}\bar{s}_{j|g} - 2s_j\right)\left(\frac{1}{1-\sigma} - \frac{\sigma}{1-\sigma}\bar{s}_{j|g} - s_j\right) s_j - \frac{\sigma}{(1-\sigma)^2} s_j \bar{s}_{j|g}\left(1 - \bar{s}_{j|g}\right)}$$
$$= \frac{(1-\sigma)s_k\left(1 - \sigma\bar{s}_{j|g} - 2(1-\sigma)s_j\right)}{\left(1 - \sigma\bar{s}_{j|g} - 2(1-\sigma)s_j\right)\left(1 - \sigma\bar{s}_{j|g} - (1-\sigma)s_j\right) - \sigma\bar{s}_{j|g}\left(1 - \bar{s}_{j|g}\right)}$$
$$= \frac{(1-\sigma)s_k}{\left(1 - \sigma\bar{s}_{j|g} - (1-\sigma)s_j\right) - \frac{\sigma\bar{s}_{j|g}\left(1-\bar{s}_{j|g}\right)}{\left(1-\sigma\bar{s}_{j|g} - 2(1-\sigma)s_j\right)}} \tag{A.6}$$

If the second term in the denominator were zero, observe that:

$$D_{jk}^{**} = \frac{(1-\sigma)s_k}{\left(1-\sigma\bar{s}_{j|g}-(1-\sigma)s_j\right)}$$

The second term is not equal to zero unless $s_{j|g} \in \{0,1\}$, so the nested logit does not appear to satisfy the curvature property. This means that diversion ratios do not exhibit constant treatment effects. We have confirmed via simulation that second-choice diversion ratios are not equal to diversion ratios measured by marginal price changes.

Random-Coefficients Logit Demand

Random-coefficients logit demand relaxes the IIA property of the plain logit model, which can be desirable empirically and leads to non-constant treatment-effects measures of diversion ratios. We can repeat the same exercise for the logit model with random coefficients:[58]

$$u_{ij} = \underbrace{V_{ij}}_{\beta_i x_j + \xi_j} + \varepsilon_{ij}$$

If we condition on the consumers' "type" (β_i, α_i) and integrate out over ε_i the individual diversion ratio admits a closed form:

$$D_{jk,i}(x) = \frac{\frac{\partial s_{ik}}{\partial Z_j}(x)}{-\frac{\partial s_{ij}}{\partial Z_j}(x)} = \frac{-\beta_i^z \cdot s_{ik}(x)\cdot s_{ij}(x)}{-\beta_i^z \cdot s_{ij}(x)\cdot(1-s_{ij}(x))} = \frac{s_{ik}(x)}{1-s_{ij}(x)}$$

For a small price change, the corresponding weight is given by $w_{ij}(x) = \frac{s_{ij}(x)\cdot(1-s_{ij}(x))\cdot\left|\beta_i^z\right|}{\int s_{ij}(x)\cdot(1-s_{ij}(x))\cdot\left|\beta_i^z\right| d F_i}$ and for second-choice data, the corresponding weight is given by $w_{ij}(x) = \frac{s_{ij}(x)}{s_j(x)}$. Thus, small price changes and second-choice data will not yield the same average diversion ratio measures when we compute $\int D_{jk,i}(x)\, w_{ij}(x) d F_i$.

As in the previous logit and nested-logit examples, we can directly attempt to verify the

[58] We can think about price as part of x_j with a corresponding β_i^p.

properties for small price changes and second-choice data. Below, we consider the case where $\beta_i^p = \beta^p$ does not have a random coefficient (though other characteristics may). We show that even in this simplified case, small price changes and second-choice data give different diversion measures.

$$\frac{\frac{\partial s_k}{\partial P_j}}{\left|\frac{\partial s_j}{\partial P_j}\right|} = \frac{\int s_{ij} s_{ik} \frac{\partial V_{ik}}{\partial P_j}}{\int s_{ij}(1-s_{ij}) \frac{\partial V_{ij}}{\partial P_j}} \qquad \rightarrow \frac{\int s_{ij} s_{ik}}{\int s_{ij}(1-s_{ij})} \tag{A.7}$$

$$\frac{S_k(\mathcal{J},x) - S_k(\mathcal{J}\setminus j, x)}{S_j(\mathcal{J},x)} = \frac{\int \frac{e^{V_{ik}}}{1+\sum_{l\in a\setminus j} e^{V_{il}}} - \frac{e^{V_{ik}}}{1+\sum_{l'\in a} e^{V_{il'}}}}{\int -\frac{e^{V_{ij}}}{1+\sum_{l\in a} e^{V_{il}}}} \qquad \rightarrow \frac{1}{s_j}\int \frac{s_{ij} s_{ik}}{(1-s_{ij})} \tag{A.8}$$

Each individual exhibits constant diversion $D_{jk,i}(x) = \frac{s_{ik}(x)}{1-s_{ij}(x)}$, but weights on individuals vary depending on how diversion is measures, so that diversion is only constant if $s_{ij}(x) = s_j(x)$. The more correlated $(s_{ij}(x), s_{ik}(x))$ are (and especially as they are correlated with β_i) the greater the discrepancy between the two measures of average diversion. The various weights are all described in Table 2 in the main text.

B. Additional Empirical Results

In this section we present some additional results from our empirical applications to the Berry et al. (1999) automobile data and the Nevo (2000b) (fake) cereal data.

In Table B.1 we present the mean taste parameters for various product characteristics. These are sometimes referred to as the β or $\overline{\beta}$ parameters. In general, they have the anticipated signs. In the automobile example, consumers prefer faster vehicles (more horsepower relative to weight), air conditioning, and size. In the cereal example, consumers prefer sugary cereals (on average) and dislike mushy cereals (on average).

In Table B.2 we compute the pairwise correlation between each of the treatment-effects parameters (price change, quality change, second-choice data) as well as the simplified models

	Best Practices	$\Sigma_{\text{cons}} = \pi_{\text{cons}} = 0$	$\Sigma_p = \pi_p = 0$	Rescaled Shares
BLP				
price/inc	-51.254	-49.175	-0.355	-11.277
	(5.847)	(7.104)	(0.030)	(2.218)
βcons	-5.372	-5.714	-9.608	-6.551
	(0.521)	(0.450)	(1.577)	(0.662)
$\beta_{\text{HP/weight}}$	3.739	3.070	4.879	1.274
	(0.810)	(0.891)	(2.746)	(0.770)
β_{air}	0.513	0.728	2.682	-0.050
	(0.332)	(0.282)	(10.265)	(0.287)
$\beta_{\text{MP\$}}$	-0.005	0.239	0.041	0.522
	(0.156)	(0.088)	(0.157)	(0.113)
β_{size}	3.614	3.567	2.585	4.209
	(0.423)	(0.438)	(0.945)	(0.242)
Nevo				
α_{price}	-31.125	-42.642	-30.939	-36.280
	(4.700)	(3.627)	(0.913)	(2.507)
β_{cons}	-2.925	0.373	-1.977	-4.165
	(0.343)	(0.331)	(0.342)	(0.337)
β_{sugar}	0.234	0.109	0.188	0.211
	(0.035)	(0.035)	(0.035)	(0.034)
β_{mushy}	-0.886	-0.970	-0.867	-0.740
	(0.439)	(0.427)	(0.436)	(0.426)

Table B.1: Linear Utility Demand Parameters

(logit and nested logit). For this exercise we consider diversion among all pairs of products. As our weights predict, the correlation between the second-choice data and small-quality changes is very high (even higher than correlation with the small price changes). The correlation between small price change diversion and small quality change diversion is much higher in the Nevo example than the BLP example and is driven by differences in the distribution of $|\alpha_i|$, which is more dispersed in the BLP example. This is a fact we highlight below in Figure B.1.

As one might also expect, the correlation between the nested-logit and logit models is high, and the correlation between the nested-logit (or logit) models and random-coefficients models (for any treatment-effects parameter) is much lower. This is again more pronounced in the BLP example than the Nevo example where there is less overall heterogeneity in the $D_{jk,i}(x)$ measures.

	$D_{jk}(p)$	Small Quality Change	Second Choice	Logit	Nested Logit $D_{jk}(p)$
$D_{jk}(p)$	1.000	0.912	0.909	0.346	0.329
Small Quality Change	0.912	1.000	0.999	0.399	0.377
Second Choice	0.909	0.999	1.000	0.394	0.372
Logit	0.346	0.399	0.394	1.000	0.916
Nested Logit $D_{jk}(p)$	0.329	0.377	0.372	0.916	1.000

	$D_{jk}(p)$	Small Quality Change	Second Choice	Logit	Nested Logit $D_{jk}(p)$
$D_{jk}(p)$	1.000	0.990	0.991	0.682	0.676
Small Quality Change	0.990	1.000	0.999	0.725	0.721
Second Choice	0.991	0.999	1.000	0.716	0.712
Logit	0.682	0.725	0.716	1.000	0.981
Nested Logit $D_{jk}(p)$	0.676	0.721	0.712	0.981	1.000

Table B.2: Pairwise Correlation Among Treatment Effects Parameters

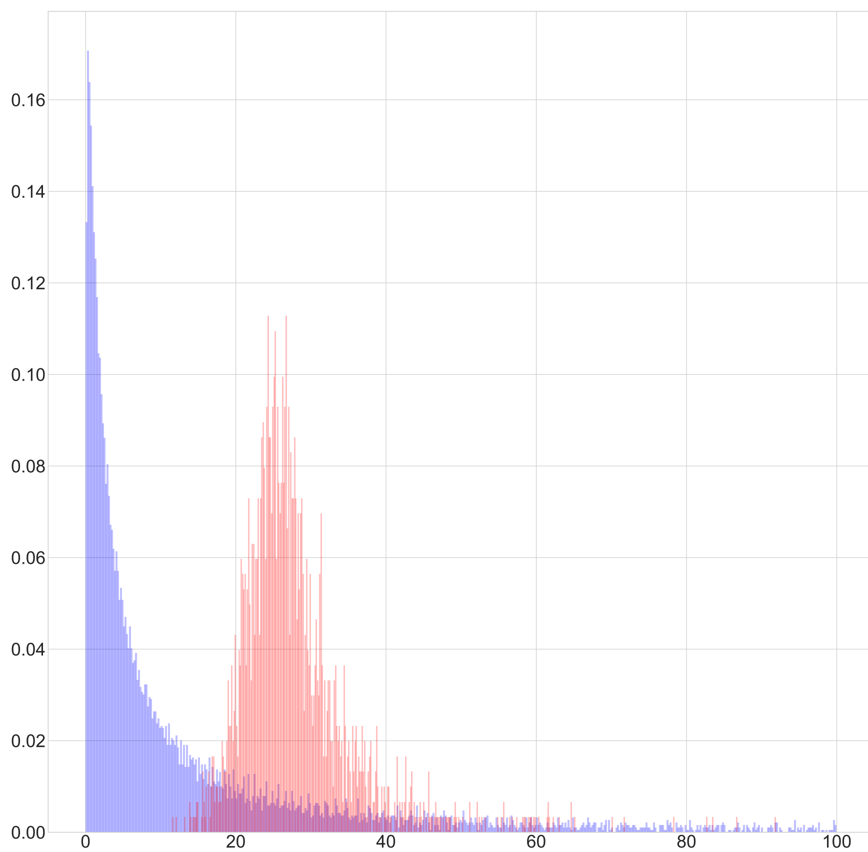


Figure B.1: Density of α_i for BLP (Blue) and Nevo (Red) Data