Supplemental Online Appendix for "Best Practices for Differentiated Products Demand Estimation with PyBLP"

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OA1. Knittel and Metaxoglou (2014) Replication

We extend our comparison of different optimization algorithms and starting values to the two example problems in Knittel and Metaxoglou (2014): the problem from Nevo (2000) and a demand-only version of the problem in Berry et al. (1995), which is a much simpler version of the full Berry et al. (1995, 1999) model replicated first in Section 6. For each example problem, we consider two configurations.

First, we use similar configurations as Knittel and Metaxoglou (2014): the same set of pseudo-Monte Carlo (pMC) draws and loose outer-loop tolerances in both problems. Knittel and Metaxoglou (2014) use a tolerance of `1E-3` for changes in the parameter vector and the objective function. Because of the loose objective function tolerance in particular, optimization routines often terminate too early when the slope of the objective function becomes less steep. We replicate this behavior with loose L^∞ gradient- and parameter-based tolerances of `1E-1`. We also set a limit of 1,000 objective evaluations, which turns out to be persistently binding only for the derivative-free Nelder-Mead routine.

Second, we slightly modify the configurations to eliminate the difficulties encountered by Knittel and Metaxoglou (2014). For the problem in Nevo (2000), it suffices to use tighter outer-loop tolerances: L^∞ gradient- and parameter-based tolerances of `1E-5`. For the demand-only Berry et al. (1995) problem, instead of 50 pMC draws in each market, we additionally use a Gauss-Hermite product rule that exactly integrates polynomials of degree 11 or less.

We use the same five optimization configurations from Table 6, along with three additional configurations considered in Online Appendix OA8: two more Knitro algorithms (Active Set and SQP) and for comparison's sake, a second TNC (truncated Newton algorithm) configuration with a gradient-based termination condition.

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For each optimization configuration and example problem, we first report convergence statistics across 50 different starting values in Table OA1. For the Nevo (2000) problem we draw starting values from a uniform distribution with support 50% above and below the starting values employed by the original paper. For the demand-only Berry et al. (1995) paper, we draw starting values for standard deviations from a uniform distribution with support on $[0.5, 1.5]$.[1]

Although the "Replication" rows in Table OA1 suggest high convergence rates, there is substantial variation in both the GMM objective value and the norm of the gradient. Loose outer-loop tolerances and instability from imprecise numerical integration create a number of spurious local minima. Under "Best Practices," most solvers converge to the same results and satisfy both first- and second-order optimality conditions.[2]

To demonstrate these results graphically, in Figures OA1 to OA5 we present histograms of own-price elasticities, markups, and GMM objective values from this replication exercise. The elasticities and markups are for the same products considered in Knittel and Metaxoglou (2014): those with the median and largest market shares. Continuing to follow the original paper, in all histograms we exclude values for routines that fail to find a local minima, which practitioners would likely discard anyways. This includes values from the Nelder-Mead routine, which essentially always failed to meet its convergence criteria after 1,000 objective evaluations.

Best estimation practices eliminate the difficulties encountered by Knittel and Metaxoglou (2014). This underscores the importance of tight optimization tolerances and the use of accurate methods for numerical integration. The impact of various other best practices (e.g., derivative-based solvers, reasonable starting values, and robust error handling) are also reflected in the "Replication" histograms, which exhibit less dispersion than their counterparts in Knittel and Metaxoglou (2014).

[1] Unlike Knittel and Metaxoglou (2014), we do not choose starting values from the standard normal distribution. Our goal is to focus on difficulties that are likely to be encountered by practitioners, who are unlikely to choose negative starting values for standard deviation parameters. However, we are able to generally replicate their findings with loose termination tolerances, and we find similar results either way.

[2] As in Section 5.2, the exception is Nelder-Mead, which is the only derivative-free solver in Table OA1.

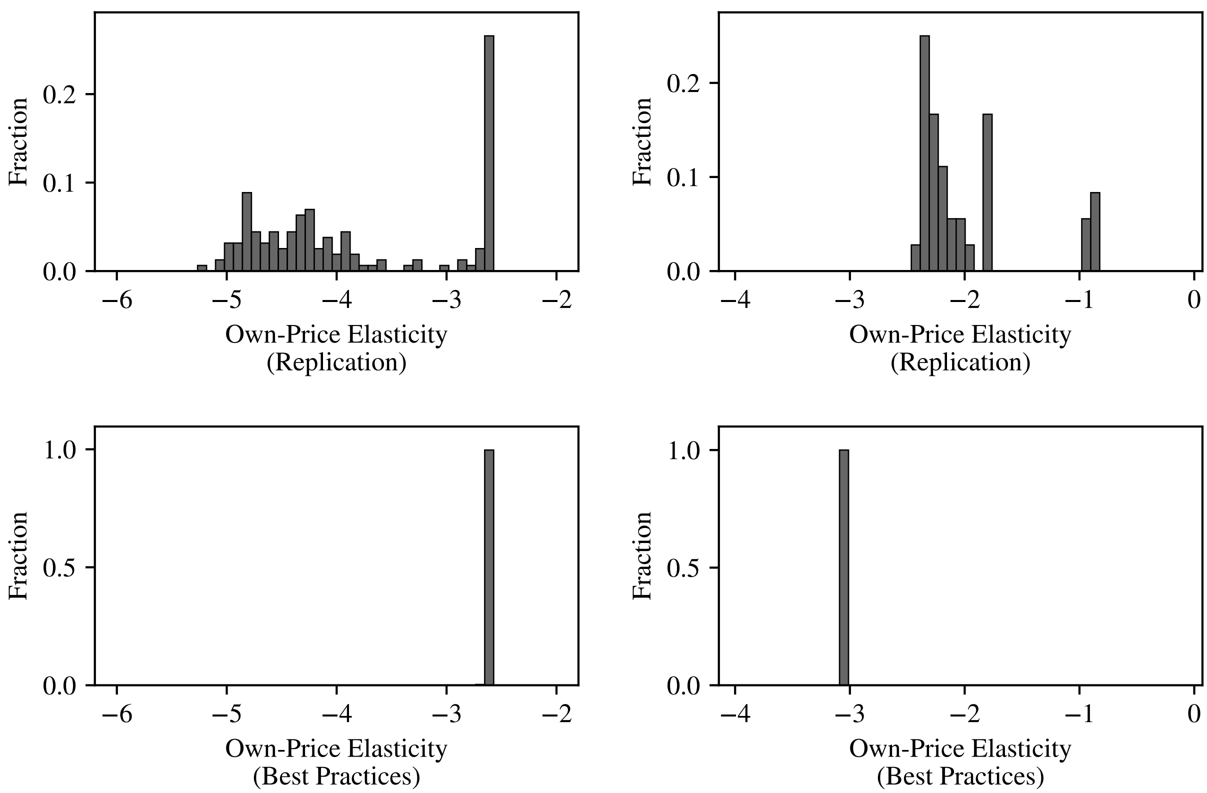
Table OA1: Optimization Algorithms: Knittel and Metaxoglou (2014) Replication

						Percent of Runs		Median, First GMM Step			
Example	$\|\theta_2\|$	Software	Algorithm	Gradient	Termination	Converged	PSD Hessian	Seconds	Evaluations	$q = \bar{g}'W\bar{g}$	$\|\|\nabla q\|\|_\infty$
Nevo: Best Practices	13	Knitro	Interior/Direct	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	67.9	114	2.02E-03	7.31E-06
Nevo: Best Practices	13	Knitro	Active Set	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	69.8	119	2.02E-03	5.60E-06
Nevo: Best Practices	13	Knitro	SQP	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	69.9	117	2.02E-03	5.87E-06
Nevo: Best Practices	13	SciPy	L-BFGS-B	Yes	$\|\|\nabla q\|\|_\infty$	42.0%	100.0%	575.0	1,002	2.02E-03	5.91E-05
Nevo: Best Practices	13	SciPy	BFGS	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	82.2	138	2.02E-03	5.68E-06
Nevo: Best Practices	13	SciPy	TNC	Yes	$\|\|\nabla q\|\|_\infty$	86.0%	100.0%	395.5	513	2.02E-03	5.61E-08
Nevo: Best Practices	13	SciPy	TNC	Yes	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	100.0%	100.0%	89.1	134	2.02E-03	2.34E-05
Nevo: Best Practices	13	SciPy	Nelder-Mead	No	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	0.0%	76.0%	437.7	1,001	8.04E-03	2.49E-02
Nevo: Replication	13	Knitro	Interior/Direct	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	26.0%	4.8	8	1.86E-02	6.77E-02
Nevo: Replication	13	Knitro	Active Set	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	28.0%	5.0	8	1.81E-02	5.83E-02
Nevo: Replication	13	Knitro	SQP	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	24.0%	4.8	8	1.86E-02	6.73E-02
Nevo: Replication	13	SciPy	L-BFGS-B	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	24.0%	3.4	5	1.95E-02	6.02E-02
Nevo: Replication	13	SciPy	BFGS	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	32.0%	3.8	6	1.88E-02	7.14E-02
Nevo: Replication	13	SciPy	TNC	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	69.7	107	2.02E-03	3.22E-04
Nevo: Replication	13	SciPy	TNC	Yes	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	100.0%	82.0%	2.9	5	1.56E-02	1.64E-01
Nevo: Replication	13	SciPy	Nelder-Mead	No	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	0.0%	78.0%	442.3	1,001	8.04E-03	2.49E-02
BLP: Best Practices	5	Knitro	Interior/Direct	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	945.5	42	1.03E-01	1.97E-06
BLP: Best Practices	5	Knitro	Active Set	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	947.5	42	1.03E-01	3.11E-06
BLP: Best Practices	5	Knitro	SQP	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	941.7	42	1.03E-01	2.06E-06
BLP: Best Practices	5	SciPy	L-BFGS-B	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	790.1	42	1.03E-01	2.28E-06
BLP: Best Practices	5	SciPy	BFGS	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	889.9	50	1.03E-01	3.43E-06
BLP: Best Practices	5	SciPy	TNC	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	100.0%	1,504.5	57	1.03E-01	2.03E-06
BLP: Best Practices	5	SciPy	TNC	Yes	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	100.0%	100.0%	1,718.6	73	1.03E-01	1.25E-06
BLP: Best Practices	5	SciPy	Nelder-Mead	No	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	0.0%	100.0%	16,515.7	1,002	1.03E-01	9.66E-08
BLP: Replication	5	Knitro	Interior/Direct	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	0.0%	9.3	6	1.32E-01	3.50E-02
BLP: Replication	5	Knitro	Active Set	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	6.0%	5.1	6	1.29E-01	3.31E-02
BLP: Replication	5	Knitro	SQP	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	4.0%	8.8	6	1.30E-01	3.07E-02
BLP: Replication	5	SciPy	L-BFGS-B	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	0.0%	2.5	4	1.33E-01	2.82E-02
BLP: Replication	5	SciPy	BFGS	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	0.0%	2.5	4	1.32E-01	3.27E-02
BLP: Replication	5	SciPy	TNC	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	8.0%	7.8	11	1.22E-01	2.49E-02
BLP: Replication	5	SciPy	TNC	Yes	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	100.0%	54.0%	10.9	27	9.86E-02	1.09E-02
BLP: Replication	5	SciPy	Nelder-Mead	No	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	0.0%	100.0%	230.0	1,002	9.23E-02	9.42E-08

This table documents the same optimization convergence statistics as Table 6 for the example problems from Knittel and Metaxoglou (2014) solved with 50 different starting values instead of across different simulated datasets. In "Replication" rows we document some of the difficulties encountered by Knittel and Metaxoglou (2014) and in "Best Practices" rows we attempt to eliminate these difficulties with best estimation practices.

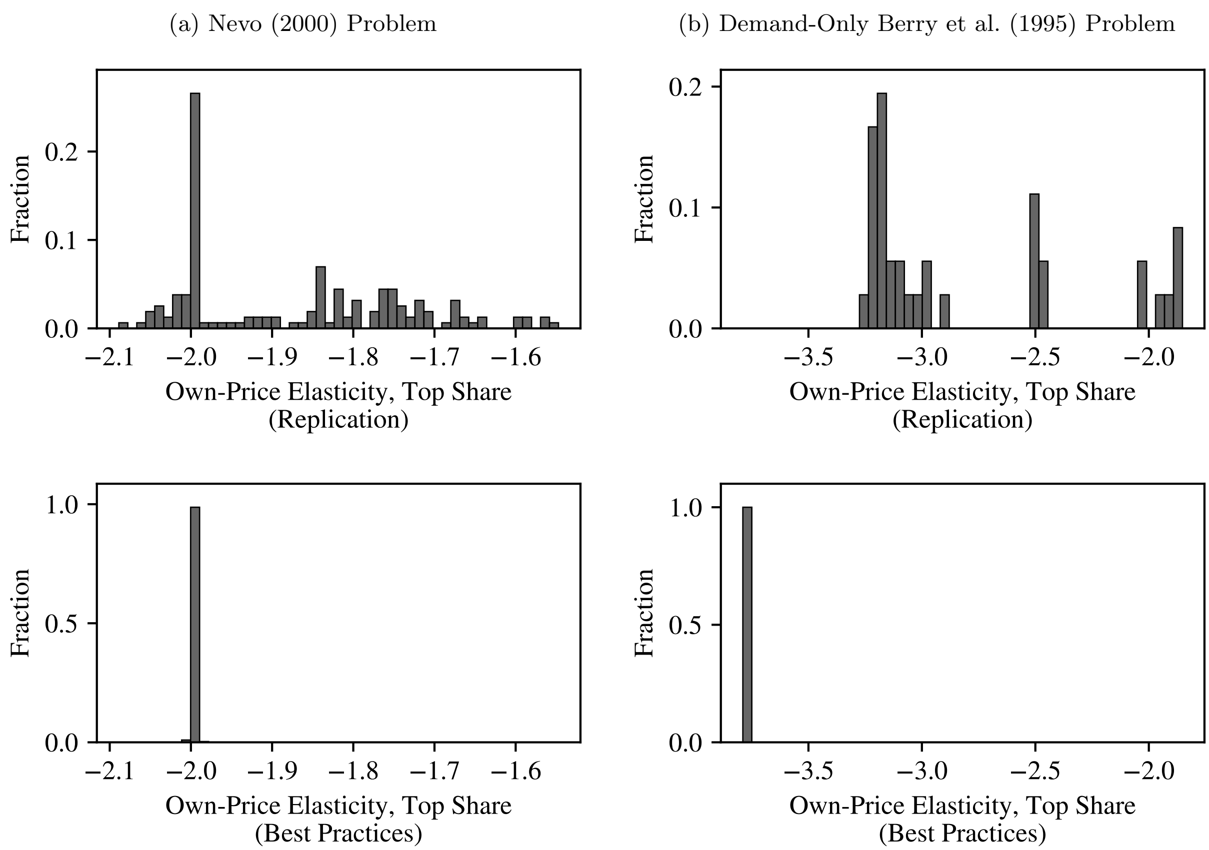
Figure OA1: Histograms for Median Product Own-Price Elasticities

(a) Nevo (2000) Problem (b) Demand-Only Berry et al. (1995) Problem



These plots document the distribution of own-price elasticities for the product with the median market share from Table OA1. Values for Nelder-Mead and other routines that failed to find a local minimum are excluded.

Figure OA2: Histograms for Top Product Own-Price Elasticities

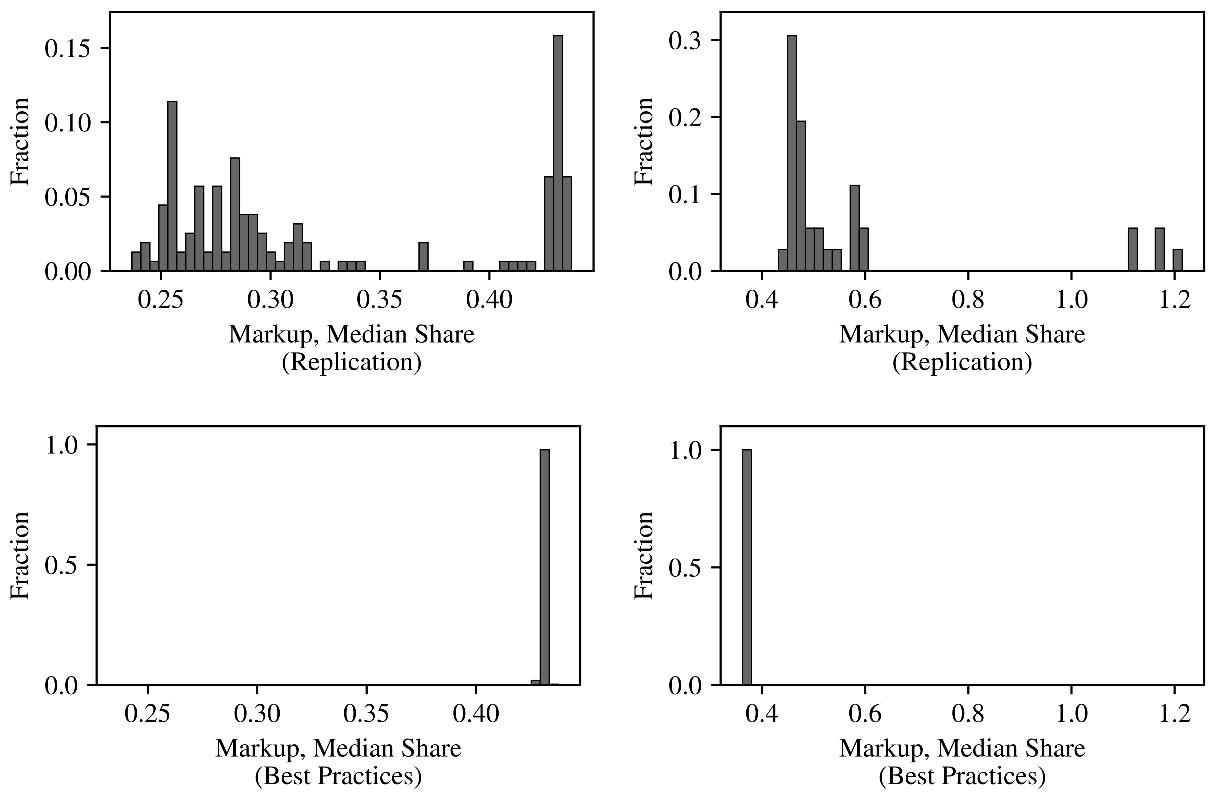


These plots document the distribution of own-price elasticities for product with the largest market share from Table OA1. Values for Nelder-Mead and other routines that failed to find a local minimum are excluded.

Figure OA3: Histograms for Median Product Markups

(a) Nevo (2000) Problem

(b) Demand-Only Berry et al. (1995) Problem

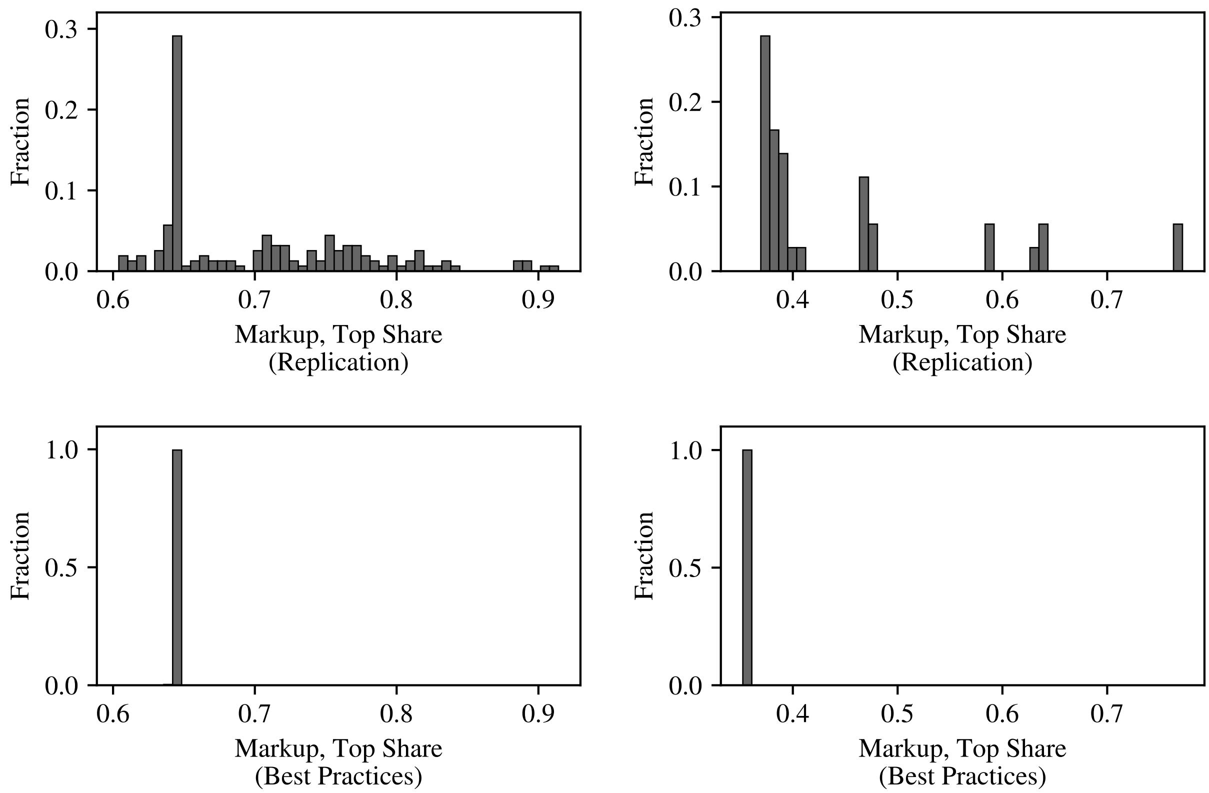


These plots document the distribution of markups for product with the median market share from Table OA1. Values for Nelder-Mead and other routines that failed to find a local minimum are excluded.

Figure OA4: Histograms for Top Product Markups

(a) Nevo (2000) Problem

(b) Demand-Only Berry et al. (1995) Problem

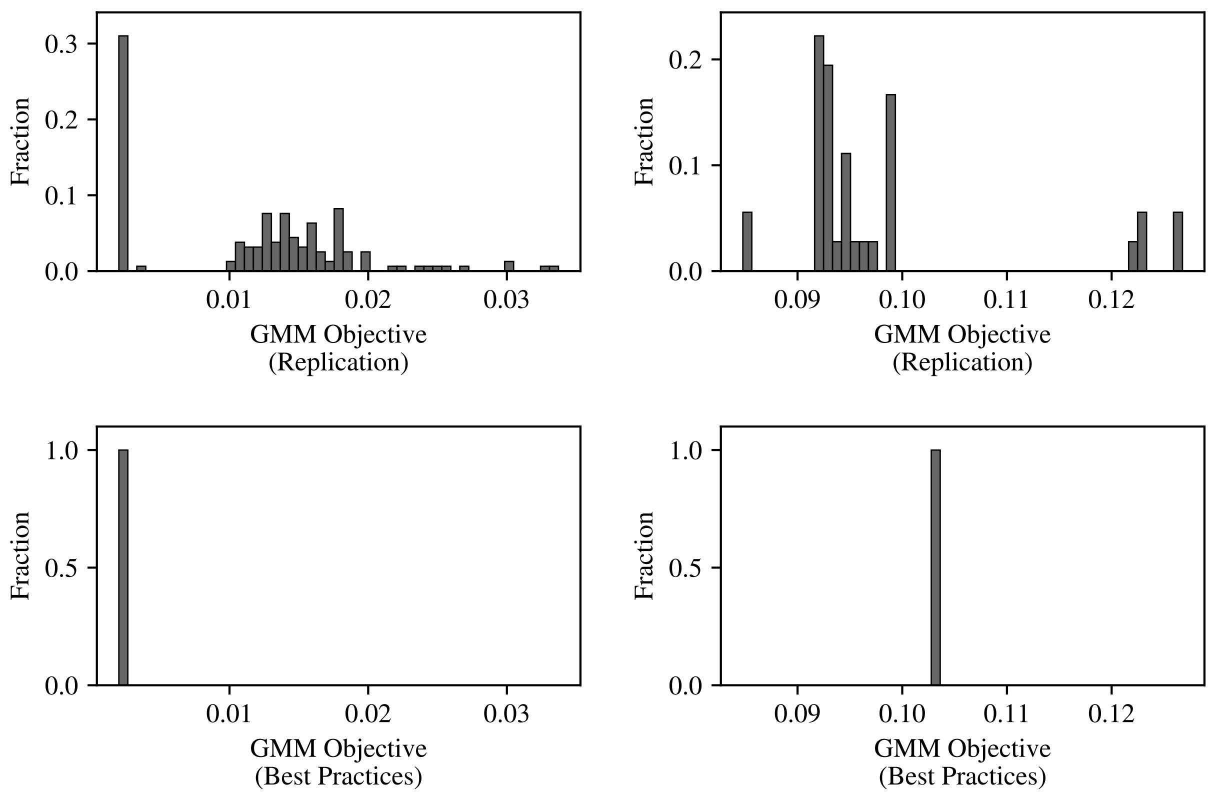


These plots document the distribution of markups for product with the largest market share from Table OA1. Values for Nelder-Mead and other routines that failed to find a local minimum are excluded.

Figure OA5: Histograms for GMM Objective Values

(a) Nevo (2000) Problem

(b) Demand-Only Berry et al. (1995) Problem



These plots document the distribution of GMM objective values from Table OA1. Values for Nelder-Mead and other routines that failed to find a local minimum are excluded.

OA2. Integration

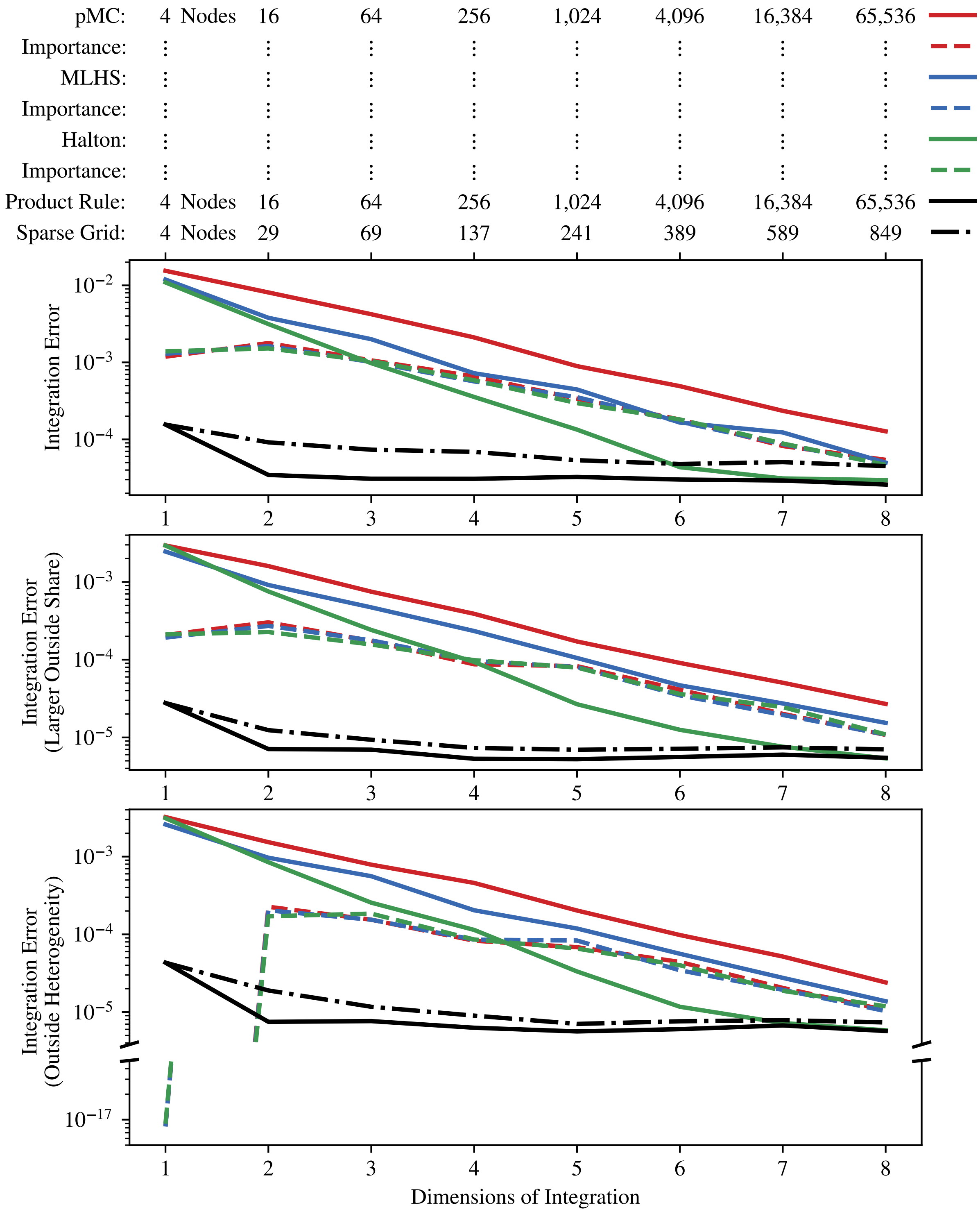
In Section 5.2 we found the benefits of importance sampling to be largely underwhelming. In Figure OA6 we study situations in which we expect importance sampling to perform better.

The top plot is the same as Figure 1 but with lines that document the performance of importance sampling when based on pseudo-Monte Carlo (pMC) and Modified Latin Hypercube Sampling (MLHS) in addition to Halton draws. Performance doesn't seem to be particularly affected by the type of nodes that are sampled from.

In the middle and bottom plots we decrease the linear parameter on the outside good from $\beta_0 = -7$ to $\beta_0 = -10$ so that the outside share increases from $s_{0t} \approx 0.9$ to $s_{0t} \approx 0.98$. In the bottom plot we additionally substitute the first random coefficient on a continuously-varying variable with a random coefficient on a constant term (i.e., we add heterogeneous preferences for the outside good). Both changes somewhat improve the performance of importance sampling, although the improvement diminishes with the number of random coefficients, and is not very robust to considering relative instead of absolute integration error (Figure OA7).

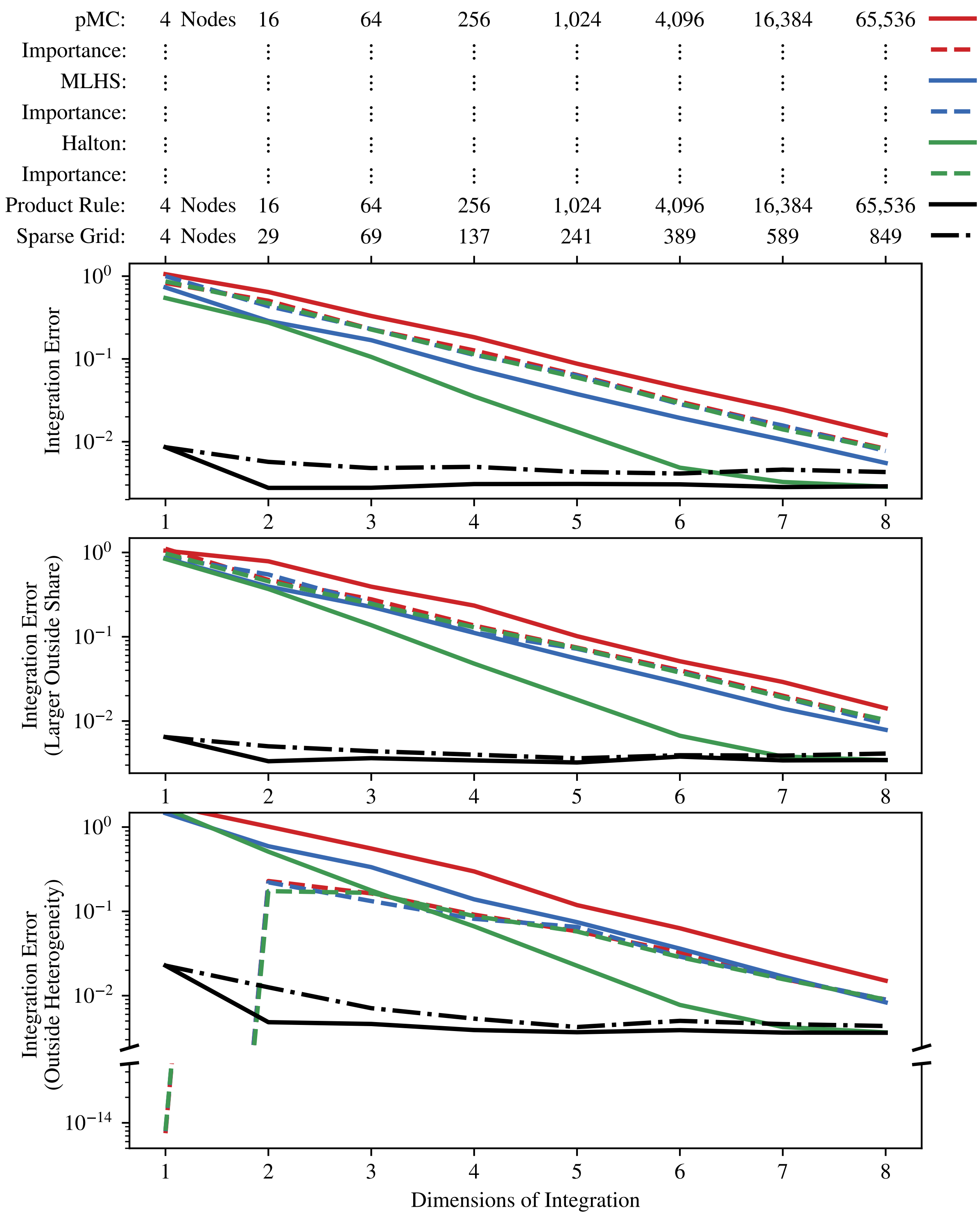
The one exception is when the only heterogeneity in the problem is for the outside good, in which case importance sampling at the true parameter values effectively eliminates all integration error. We should caution that in practice, importance sampling requires a good estimate $\widetilde{\theta}_2$. In these plots we consider the best-case scenario with the true parameter values.

Figure OA6: Integration Error: Importance Sampling



The top plot is the same as Figure 1 but with additional lines for importance sampling based on pseudo-Monte Carlo (pMC) and Modified Latin Hypercube Sampling (MLHS). The middle and bottom plots are based on simulations with a larger outside share of $s_{0t} \approx 0.98$ instead of $s_{0t} \approx 0.9$. In the bottom plot, we additionally replace the first random coefficient with a random coefficient on the constant term so that there are heterogeneous preferences for the outside good. In this special case we break the y-axis to document that at the true parameter values $\widetilde{\theta}_2$, importance sampling reduces the integration error to essentially zero.

Figure OA7: Integration Error: Importance Sampling and Relative RMSE



These plots are the same as those in Figure OA6 but report relative root mean square error (RMSE) of market shares instead of absolute RMSE.

OA3. Instruments

We report some additional results on instrument strength, misspecification, and the impact of instrument choice on the shape of the GMM objective function.

In Figure OA8 we document the impact of instrument strength and misspecification on the variance of $\hat{\alpha}$. Like the bias in Figure 2, variance decreases as the cost shifter becomes more strongly correlated with prices, a well-specified supply side further reduces variance, and a misspecified supply side blows up the variance of the estimator, especially when the cost shifter is weak.

Similarly, Figure OA9 is the variance analogue of Figure A1 where we document how supply-side moments can improve the construction of feasible optimal instruments for demand-only problems. Using supply moments to construct feasible optimal instruments helps to reduce the variance of demand-only estimates when the supply side is well-specified, but under misspecification slightly increases the variance of estimates, especially when the cost shifter is weak.

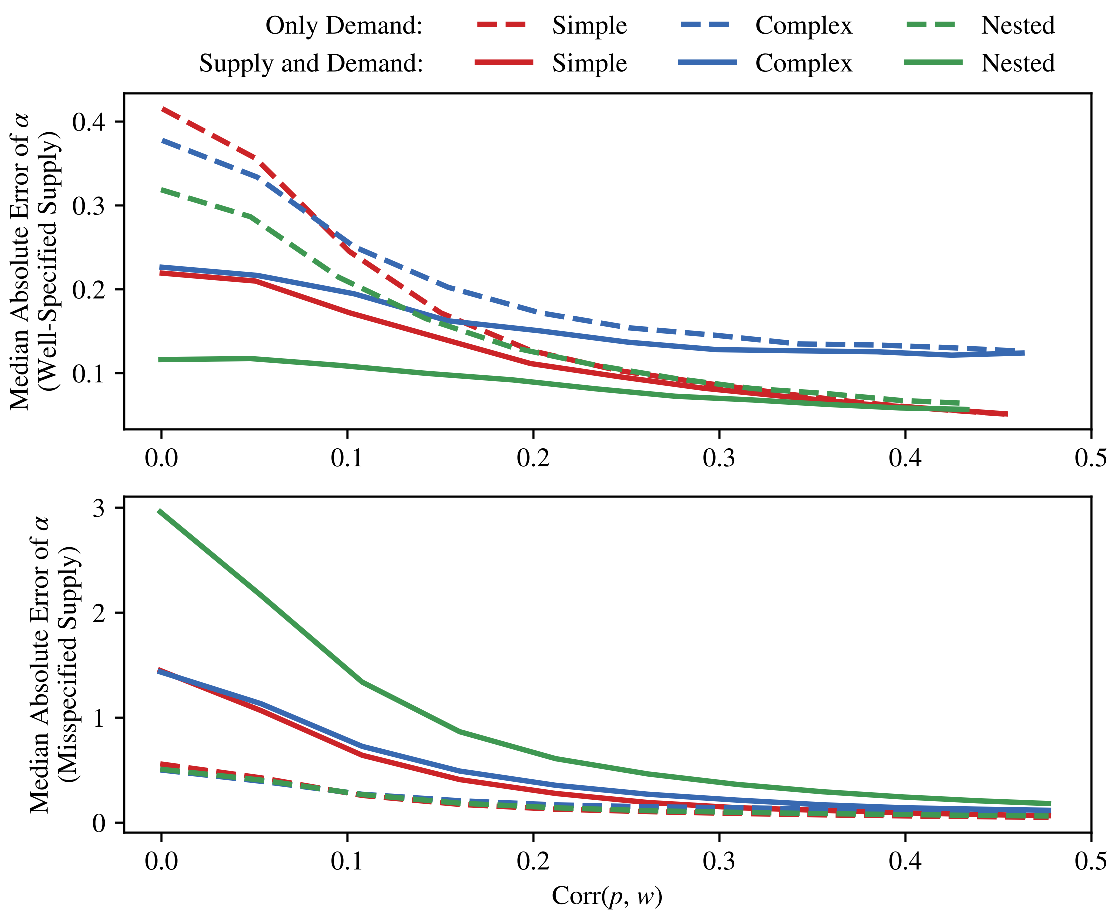
In Figures OA10 and OA11 we replicate the results from Figures 2 and OA8 without feasible optimal instruments. Instead, we use sums of characteristics BLP instruments. The qualitative conclusions are similar. Quantitatively, however, bias and variance are reduced less when supply side moments are added. On the other hand, without feasible optimal instruments the problems from misspecification are less serious.

In Tables OA2 to OA5, we report bias and variance for parameters other than α. Although less stark than the improvements for α, a stronger cost shifter and a well-specified supply side also help to reduce bias and variance of other parameters.

To measure goodness of fit, we also report the J-statistic of Hansen (1982) and the LR statistic described at the end of Section 4. At standard levels of significance, the LR statistic fails to reject the well-specified supply side and rejects the misspecified model. The misspecified model is more strongly rejected when the cost shifter is stronger, and when optimal instruments are employed.

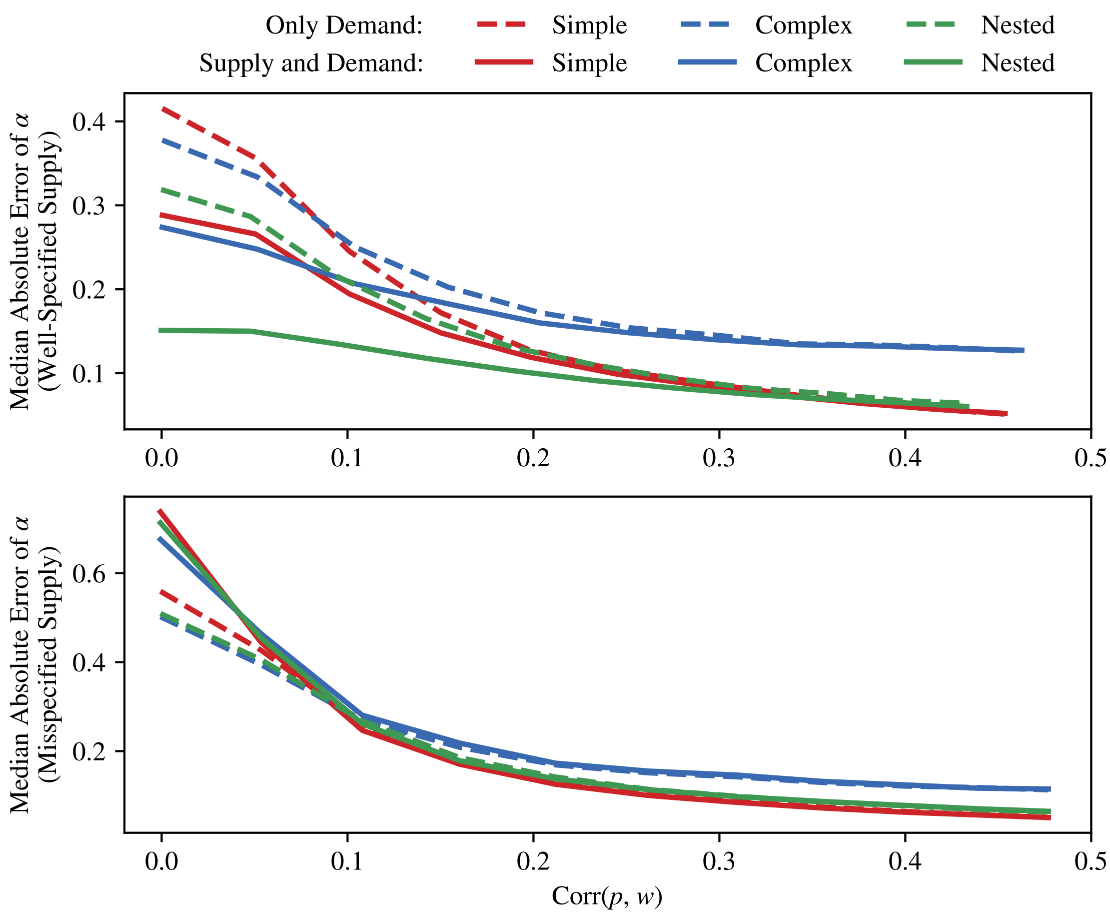
In Figures OA12 and OA13 we profile the GMM objective function while holding fixed parameters in the Complex and Nested simulation configurations. Similar to results for the Simple configuration in Figure 3, optimal instruments and the inclusion of supply moments generally make the objective steeper around the minimum.

Figure OA8: Instrument Strength and Misspecification: Variance



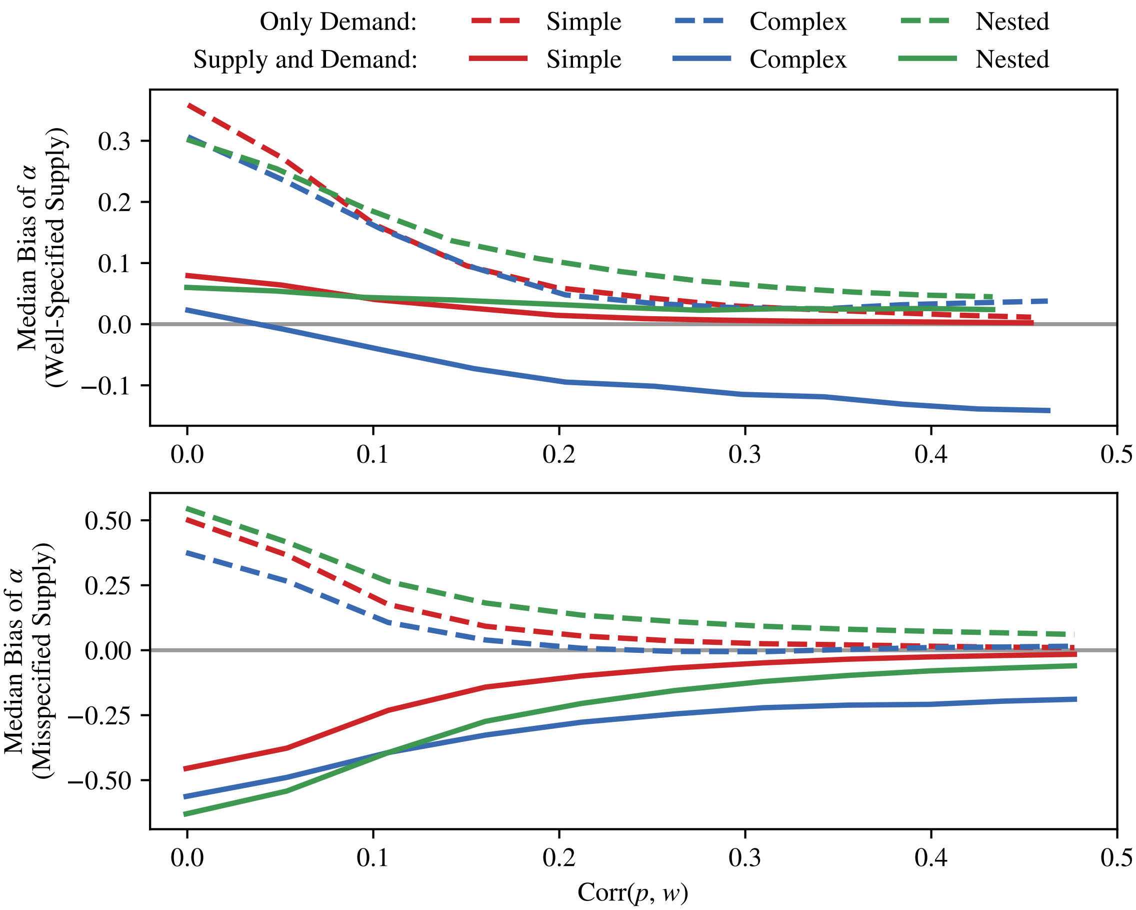
These plots are the same as those in Figure 2 but report mean absolute error instead of bias.

Figure OA9: Impact of Supply-Side Moments on Optimal IV for Demand-Only Problem: Variance



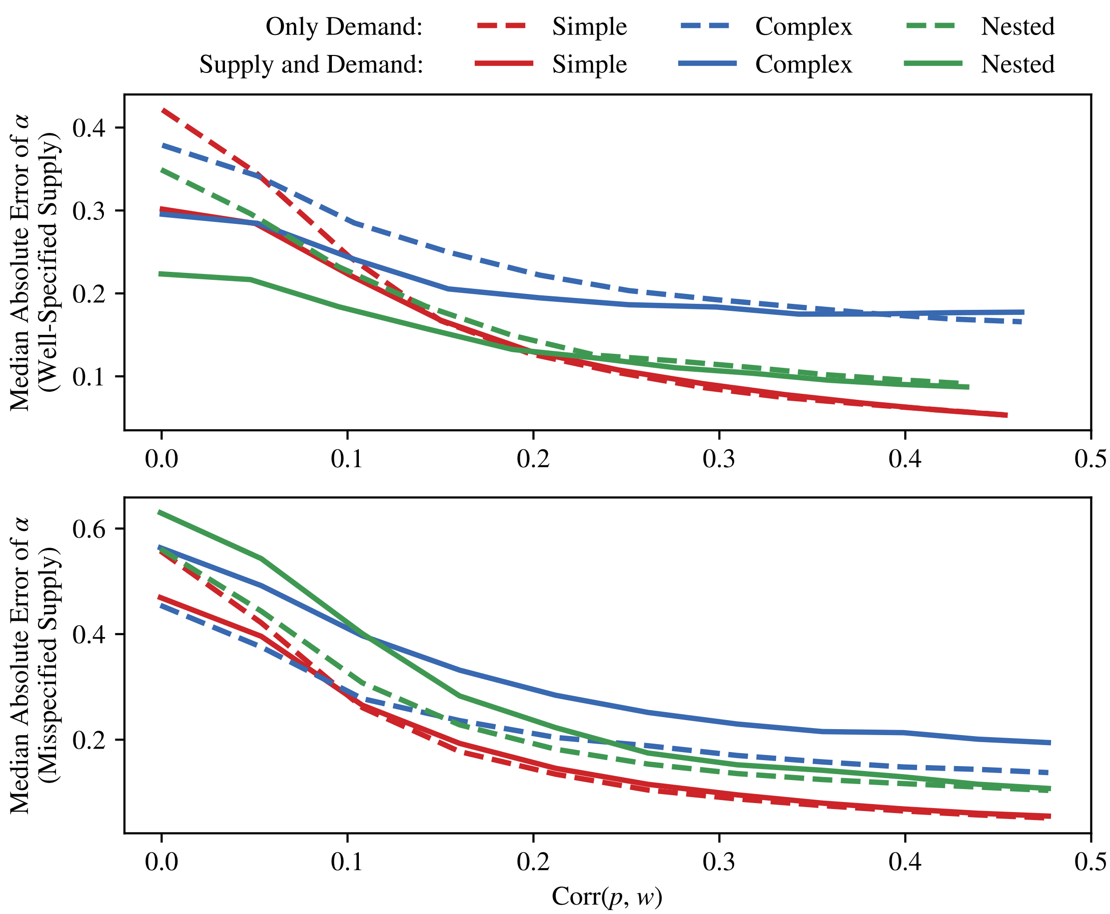
These plots are the same as those in Figure A1 but report mean absolute error instead of bias.

Figure OA10: Instrument Strength and Misspecification: Non-Optimal Instruments, Bias



These plots are the same as those in Figure 2 but without optimal instruments. Sums of characteristics BLP instruments are used instead.

Figure OA11: Instrument Strength and Misspecification: Non-Optimal Instruments, Variance



These plots are the same as those in Figure OA8 but without optimal instruments. Sums of characteristics BLP instruments are used instead.

Table OA2: Instrument Strength: Well-Specified Supply

							True Value				Median Bias				Median Absolute Error			
Simulation	γ_w	Corr(p, w)	Supply	J	LR	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	0.0	0.000	No	0.00		0.8	-1	3			0.379	-0.087			0.415	0.198		
Simple	0.0	0.000	Yes	1.94	1.94	2.3	-1	3			0.021	0.010			0.219	0.181		
Simple	0.1	0.051	No	0.00		0.8	-1	3			0.311	-0.069			0.356	0.182		
Simple	0.1	0.051	Yes	2.00	2.00	2.2	-1	3			0.018	0.007			0.210	0.178		
Simple	0.2	0.101	No	0.00		0.8	-1	3			0.189	-0.039			0.245	0.169		
Simple	0.2	0.101	Yes	2.04	2.04	2.2	-1	3			0.015	0.003			0.172	0.172		
Simple	0.4	0.198	No	0.00		0.8	-1	3			0.070	-0.006			0.127	0.165		
Simple	0.4	0.198	Yes	2.21	2.21	2.2	-1	3			-0.001	0.005			0.112	0.157		
Simple	0.8	0.377	No	0.00		0.8	-1	3			0.019	0.002			0.065	0.163		
Simple	0.8	0.377	Yes	2.29	2.29	2.5	-1	3			-0.000	0.002			0.063	0.149		
Complex	0.0	0.000	No	0.00		1.6	-1	3	0.2		0.325	-0.148	-0.010		0.378	0.204	0.176	
Complex	0.0	0.000	Yes	2.56	2.54	4.8	-1	3	0.2		-0.015	-0.036	-0.013		0.226	0.176	0.199	
Complex	0.1	0.052	No	0.00		1.6	-1	3	0.2		0.277	-0.121	-0.012		0.334	0.188	0.173	
Complex	0.1	0.052	Yes	2.68	2.66	4.7	-1	3	0.2		-0.019	-0.038	-0.020		0.217	0.180	0.199	
Complex	0.2	0.104	No	0.00		1.6	-1	3	0.2		0.173	-0.090	-0.008		0.251	0.176	0.167	
Complex	0.2	0.104	Yes	2.73	2.70	4.7	-1	3	0.2		-0.019	-0.033	0.004		0.195	0.170	0.167	
Complex	0.4	0.203	No	0.00		1.6	-1	3	0.2		0.059	-0.042	-0.020		0.172	0.166	0.164	
Complex	0.4	0.203	Yes	2.69	2.65	4.8	-1	3	0.2		-0.030	-0.029	0.001		0.151	0.160	0.165	
Complex	0.8	0.385	No	0.00		1.6	-1	3	0.2		0.023	-0.025	-0.016		0.134	0.171	0.148	
Complex	0.8	0.385	Yes	2.76	2.70	5.2	-1	3	0.2		-0.022	-0.031	0.011		0.126	0.158	0.141	
RCNL	0.0	-0.000	No	0.00		4.2	-1	3		0.5	0.282	-0.022		-0.013	0.319	0.159		0.024
RCNL	0.0	-0.000	Yes	2.96	2.96	9.4	-1	3		0.5	0.018	-0.014		0.001	0.116	0.144		0.018
RCNL	0.1	0.048	No	0.00		4.2	-1	3		0.5	0.249	-0.024		-0.011	0.287	0.155		0.023
RCNL	0.1	0.048	Yes	2.90	2.90	9.5	-1	3		0.5	0.014	-0.007		0.001	0.117	0.140		0.017
RCNL	0.2	0.095	No	0.00		4.3	-1	3		0.5	0.176	-0.017		-0.007	0.214	0.153		0.021
RCNL	0.2	0.095	Yes	2.89	2.89	9.3	-1	3		0.5	0.002	-0.001		0.000	0.109	0.139		0.018
RCNL	0.4	0.189	No	0.00		4.4	-1	3		0.5	0.079	-0.018		-0.002	0.131	0.159		0.019
RCNL	0.4	0.189	Yes	2.98	2.98	9.4	-1	3		0.5	0.001	0.000		0.000	0.092	0.140		0.016
RCNL	0.8	0.359	No	0.00		4.4	-1	3		0.5	0.029	0.005		0.000	0.076	0.155		0.019
RCNL	0.8	0.359	Yes	2.98	2.98	9.3	-1	3		0.5	-0.001	0.005		0.000	0.063	0.138		0.017

This table documents bias and variance of parameter estimates over the 1,000 simulated datasets with well-specified supply sides used to create the top plot in Figure 2. To increase the strength of the cost shifter w_{jt}, we increase its coefficient γ_w in the equation for marginal costs and report the increasing correlation between w_{jt} and price p_{jt}s. We also report J and LR test statistics to measure goodness of fit. The J statistic is zero without supply because with feasible optimal instruments the problem is just-identified.

Table OA3: Instrument Strength: Misspecified Supply

							True Value				Median Bias				Median Absolute Error			
Simulation	γ_w	Corr(p, w)	Supply	J	LR	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	0.0	-0.001	No	0.00		0.9	-1	3			0.520	0.014			0.559	0.173		
Simple	0.0	-0.001	Yes	7.28	7.28	2.6	-1	3			-1.449	0.017			1.449	0.266		
Simple	0.1	0.054	No	0.00		0.9	-1	3			0.373	0.011			0.427	0.169		
Simple	0.1	0.054	Yes	8.38	8.38	2.6	-1	3			-1.067	0.022			1.067	0.244		
Simple	0.2	0.108	No	0.00		0.9	-1	3			0.205	0.012			0.259	0.171		
Simple	0.2	0.108	Yes	10.38	10.38	2.5	-1	3			-0.642	0.017			0.642	0.212		
Simple	0.4	0.212	No	0.00		0.9	-1	3			0.067	0.029			0.128	0.175		
Simple	0.4	0.212	Yes	12.60	12.60	2.5	-1	3			-0.278	0.006			0.278	0.180		
Simple	0.8	0.399	No	0.00		0.9	-1	3			0.019	0.049			0.064	0.180		
Simple	0.8	0.399	Yes	13.51	13.51	2.7	-1	3			-0.086	-0.006			0.094	0.175		
Complex	0.0	-0.001	No	0.00		1.6	-1	3	0.2		0.452	-0.057	-0.050		0.502	0.175	0.200	
Complex	0.0	-0.001	Yes	7.59	7.51	4.6	-1	3	0.2		-1.436	-0.044	-0.200		1.436	0.279	0.200	
Complex	0.1	0.054	No	0.00		1.6	-1	3	0.2		0.344	-0.052	-0.066		0.394	0.163	0.200	
Complex	0.1	0.054	Yes	8.69	8.61	4.5	-1	3	0.2		-1.133	-0.039	-0.200		1.133	0.241	0.200	
Complex	0.2	0.108	No	0.00		1.6	-1	3	0.2		0.184	-0.036	-0.055		0.270	0.165	0.200	
Complex	0.2	0.108	Yes	10.62	10.51	4.5	-1	3	0.2		-0.724	-0.042	-0.099		0.726	0.215	0.200	
Complex	0.4	0.212	No	0.00		1.6	-1	3	0.2		0.053	-0.033	-0.054		0.169	0.181	0.200	
Complex	0.4	0.212	Yes	13.04	12.96	4.6	-1	3	0.2		-0.354	-0.044	-0.002		0.356	0.194	0.200	
Complex	0.8	0.399	No	0.00		1.7	-1	3	0.2		0.008	-0.028	-0.037		0.122	0.178	0.200	
Complex	0.8	0.399	Yes	13.24	13.14	5.3	-1	3	0.2		-0.130	-0.060	0.028		0.142	0.187	0.200	
RCNL	0.0	-0.001	No	0.00		4.8	-1	3		0.5	0.495	-0.002		0.002	0.509	0.164		0.019
RCNL	0.0	-0.001	Yes	11.05	11.05	13.0	-1	3		0.5	-2.960	0.046		0.020	2.960	0.376		0.045
RCNL	0.1	0.054	No	0.00		4.8	-1	3		0.5	0.388	-0.000		0.001	0.406	0.158		0.019
RCNL	0.1	0.054	Yes	13.25	13.25	12.7	-1	3		0.5	-2.163	0.069		0.017	2.163	0.318		0.038
RCNL	0.2	0.108	No	0.00		4.8	-1	3		0.5	0.248	0.002		0.003	0.268	0.155		0.019
RCNL	0.2	0.108	Yes	17.94	17.94	12.2	-1	3		0.5	-1.338	0.101		0.014	1.338	0.245		0.031
RCNL	0.4	0.212	No	0.00		4.8	-1	3		0.5	0.093	-0.001		0.003	0.142	0.166		0.020
RCNL	0.4	0.212	Yes	24.33	24.33	11.6	-1	3		0.5	-0.609	0.176		0.004	0.609	0.240		0.025
RCNL	0.8	0.399	No	0.00		4.9	-1	3		0.5	0.029	0.006		0.002	0.077	0.161		0.019
RCNL	0.8	0.399	Yes	28.88	28.88	11.1	-1	3		0.5	-0.244	0.230		-0.010	0.244	0.255		0.022

This table documents bias and variance of parameter estimates over the 1,000 simulated datasets with misspecified supply sides used to create the bottom plot in Figure 2. To increase the strength of the cost shifter w_{jt}, we increase its coefficient γ_w in the equation for marginal costs and report the increasing correlation between w_{jt} and prices p_{jt}. We also report J and LR test statistics to measure goodness of fit. The J statistic is zero without supply because with feasible optimal instruments the problem is just-identified.

Table OA4: Instrument Strength: Well-Specified Supply, Non-Optimal Instruments

							True Value				Median Bias				Median Absolute Error			
Simulation	γ_w	Corr(p, w)	Supply	J	LR	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	0.0	0.000	No	1.89		0.6	-1	3			0.359	-0.051			0.421	0.422		
Simple	0.0	0.000	Yes	6.47	3.93	1.6	-1	3			0.079	-0.039			0.302	0.412		
Simple	0.1	0.051	No	2.22		0.6	-1	3			0.273	-0.042			0.346	0.419		
Simple	0.1	0.051	Yes	6.54	3.67	1.6	-1	3			0.064	-0.054			0.284	0.411		
Simple	0.2	0.101	No	2.49		0.6	-1	3			0.162	-0.028			0.244	0.399		
Simple	0.2	0.101	Yes	6.64	3.44	1.5	-1	3			0.040	-0.025			0.222	0.406		
Simple	0.4	0.198	No	2.55		0.6	-1	3			0.060	-0.017			0.127	0.401		
Simple	0.4	0.198	Yes	6.77	3.47	1.6	-1	3			0.015	-0.030			0.130	0.394		
Simple	0.8	0.377	No	2.54		0.6	-1	3			0.019	-0.033			0.067	0.386		
Simple	0.8	0.377	Yes	6.89	3.45	1.8	-1	3			0.004	-0.052			0.068	0.379		
Complex	0.0	0.000	No	3.92		1.7	-1	3	0.2		0.307	-0.281	-0.200		0.379	0.618	0.200	
Complex	0.0	0.000	Yes	10.32	5.90	3.9	-1	3	0.2		0.023	-0.311	0.069		0.295	0.495	0.199	
Complex	0.1	0.052	No	4.16		1.7	-1	3	0.2		0.236	-0.296	-0.200		0.342	0.605	0.200	
Complex	0.1	0.052	Yes	10.26	5.69	3.9	-1	3	0.2		-0.008	-0.324	0.071		0.285	0.521	0.192	
Complex	0.2	0.104	No	4.38		1.7	-1	3	0.2		0.156	-0.238	-0.200		0.285	0.592	0.200	
Complex	0.2	0.104	Yes	10.33	5.47	3.9	-1	3	0.2		-0.041	-0.344	0.086		0.241	0.548	0.191	
Complex	0.4	0.203	No	4.34		1.7	-1	3	0.2		0.048	-0.231	-0.198		0.222	0.614	0.200	
Complex	0.4	0.203	Yes	10.45	5.41	3.9	-1	3	0.2		-0.095	-0.399	0.101		0.195	0.564	0.198	
Complex	0.8	0.385	No	4.47		1.8	-1	3	0.2		0.032	-0.216	-0.147		0.175	0.593	0.200	
Complex	0.8	0.385	Yes	10.32	5.45	4.4	-1	3	0.2		-0.131	-0.441	0.115		0.175	0.593	0.189	
RCNL	0.0	-0.000	No	2.75		3.2	-1	3		0.5	0.302	-0.371		0.005	0.349	0.720		0.039
RCNL	0.0	-0.000	Yes	9.01	5.72	6.4	-1	3		0.5	0.060	-0.310		0.011	0.223	0.670		0.037
RCNL	0.1	0.048	No	2.79		3.3	-1	3		0.5	0.254	-0.364		0.004	0.296	0.676		0.037
RCNL	0.1	0.048	Yes	9.08	5.69	6.5	-1	3		0.5	0.054	-0.310		0.011	0.217	0.660		0.036
RCNL	0.2	0.095	No	2.91		3.3	-1	3		0.5	0.190	-0.355		0.009	0.232	0.654		0.036
RCNL	0.2	0.095	Yes	9.06	5.54	6.3	-1	3		0.5	0.044	-0.309		0.009	0.184	0.631		0.034
RCNL	0.4	0.189	No	3.02		3.3	-1	3		0.5	0.107	-0.351		0.012	0.149	0.629		0.036
RCNL	0.4	0.189	Yes	9.30	5.60	6.3	-1	3		0.5	0.034	-0.312		0.008	0.132	0.626		0.034
RCNL	0.8	0.359	No	3.02		3.3	-1	3		0.5	0.052	-0.324		0.013	0.102	0.587		0.036
RCNL	0.8	0.359	Yes	9.38	5.76	6.4	-1	3		0.5	0.025	-0.306		0.007	0.095	0.606		0.033

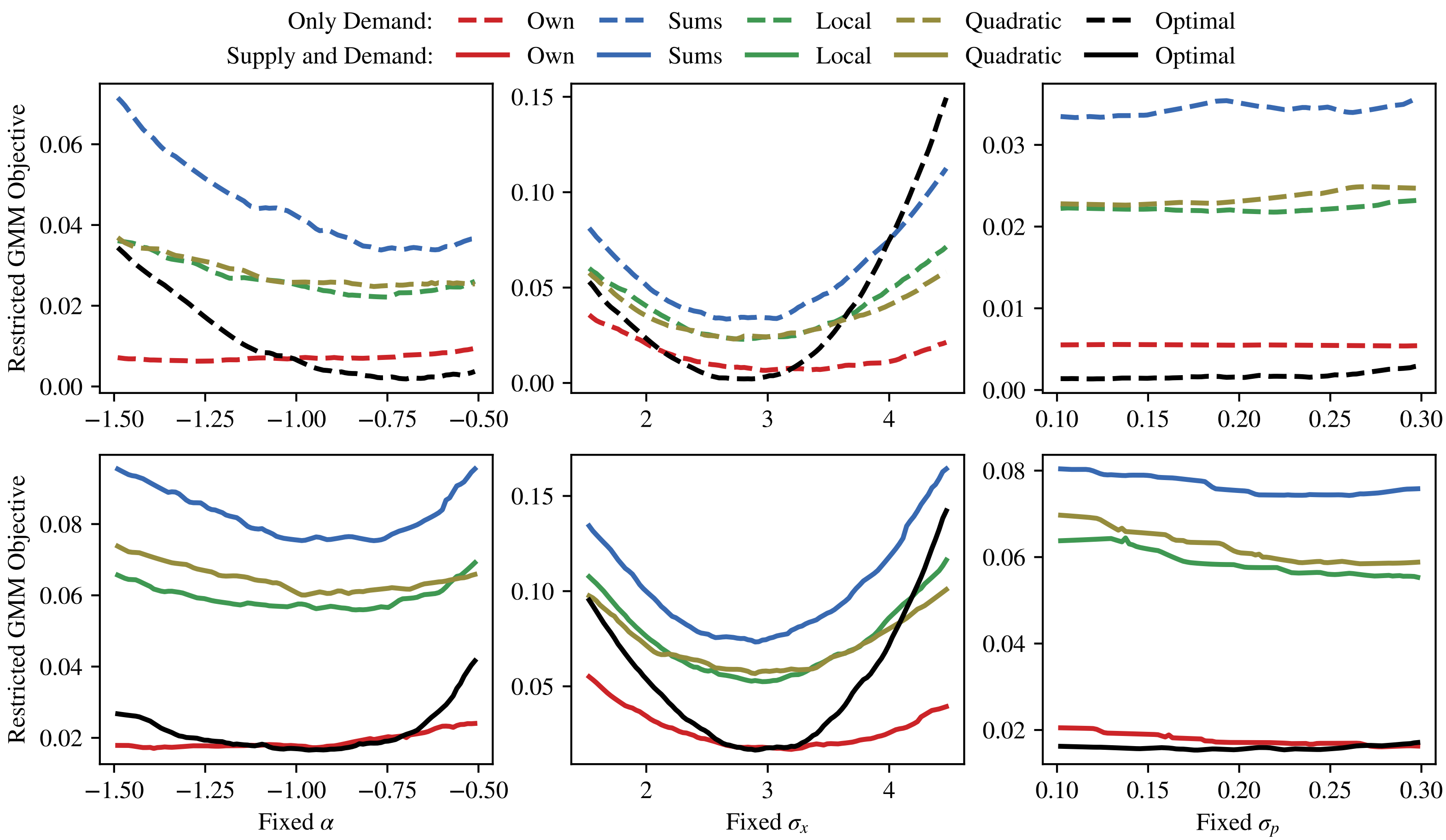
This table documents bias and variance of parameter estimates over the 1,000 simulated datasets with well-specified supply sides used to create the top plot in Figure OA10. To increase the strength of the cost shifter w_{jt}, we increase its coefficient γ_w in the equation for marginal costs and report the increasing correlation between w_{jt} and prices. We also report J and LR test statistics to measure goodness of fit.

Table OA5: Instrument Strength: Misspecified Supply, Non-Optimal Instruments

							True Value				Median Bias				Median Absolute Error			
Simulation	γ_w	Corr(p, w)	Supply	J	LR	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	0.0	-0.001	No	1.51		0.6	-1	3			0.503	-0.073			0.558	0.346		
Simple	0.0	-0.001	Yes	9.73	7.59	1.7	-1	3			-0.455	-0.134			0.469	0.429		
Simple	0.1	0.054	No	1.91		0.6	-1	3			0.366	-0.060			0.421	0.348		
Simple	0.1	0.054	Yes	10.27	7.57	1.7	-1	3			-0.377	-0.143			0.396	0.411		
Simple	0.2	0.108	No	2.37		0.6	-1	3			0.176	-0.040			0.261	0.361		
Simple	0.2	0.108	Yes	10.51	7.37	1.7	-1	3			-0.232	-0.138			0.265	0.410		
Simple	0.4	0.212	No	2.47		0.7	-1	3			0.055	-0.035			0.135	0.378		
Simple	0.4	0.212	Yes	10.71	7.53	1.8	-1	3			-0.099	-0.154			0.146	0.402		
Simple	0.8	0.399	No	2.49		0.6	-1	3			0.016	-0.013			0.065	0.409		
Simple	0.8	0.399	Yes	10.67	7.45	1.9	-1	3			-0.026	-0.165			0.069	0.417		
Complex	0.0	-0.001	No	3.19		1.8	-1	3	0.2		0.376	-0.308	-0.089		0.455	0.543	0.200	
Complex	0.0	-0.001	Yes	14.46	10.56	3.4	-1	3	0.2		-0.563	-0.479	0.200		0.564	0.583	0.200	
Complex	0.1	0.054	No	3.77		1.8	-1	3	0.2		0.266	-0.304	-0.143		0.376	0.545	0.200	
Complex	0.1	0.054	Yes	14.53	10.19	3.4	-1	3	0.2		-0.489	-0.487	0.210		0.492	0.573	0.210	
Complex	0.2	0.108	No	4.27		1.7	-1	3	0.2		0.107	-0.267	-0.198		0.278	0.521	0.200	
Complex	0.2	0.108	Yes	14.97	10.02	3.5	-1	3	0.2		-0.394	-0.487	0.205		0.397	0.561	0.205	
Complex	0.4	0.212	No	4.35		1.7	-1	3	0.2		0.008	-0.294	-0.199		0.204	0.564	0.200	
Complex	0.4	0.212	Yes	15.25	10.29	3.6	-1	3	0.2		-0.277	-0.492	0.204		0.284	0.568	0.204	
Complex	0.8	0.399	No	4.34		1.7	-1	3	0.2		0.010	-0.256	-0.200		0.148	0.572	0.200	
Complex	0.8	0.399	Yes	15.41	10.25	4.0	-1	3	0.2		-0.208	-0.488	0.183		0.213	0.559	0.200	
RCNL	0.0	-0.001	No	2.39		3.5	-1	3		0.5	0.546	-0.372		0.024	0.562	0.781		0.043
RCNL	0.0	-0.001	Yes	13.50	10.38	7.1	-1	3		0.5	-0.630	-0.334		0.036	0.630	0.932		0.058
RCNL	0.1	0.054	No	2.77		3.5	-1	3		0.5	0.416	-0.330		0.025	0.443	0.747		0.043
RCNL	0.1	0.054	Yes	13.83	10.77	7.1	-1	3		0.5	-0.542	-0.300		0.034	0.543	0.930		0.056
RCNL	0.2	0.108	No	3.00		3.5	-1	3		0.5	0.264	-0.319		0.024	0.308	0.754		0.041
RCNL	0.2	0.108	Yes	14.98	11.28	7.0	-1	3		0.5	-0.394	-0.274		0.024	0.401	0.915		0.051
RCNL	0.4	0.212	No	3.12		3.5	-1	3		0.5	0.135	-0.299		0.026	0.182	0.761		0.042
RCNL	0.4	0.212	Yes	15.87	12.29	6.8	-1	3		0.5	-0.205	-0.245		0.013	0.223	0.991		0.047
RCNL	0.8	0.399	No	3.02		3.5	-1	3		0.5	0.073	-0.298		0.024	0.117	0.747		0.040
RCNL	0.8	0.399	Yes	16.53	12.80	6.7	-1	3		0.5	-0.080	-0.232		-0.001	0.130	1.000		0.046

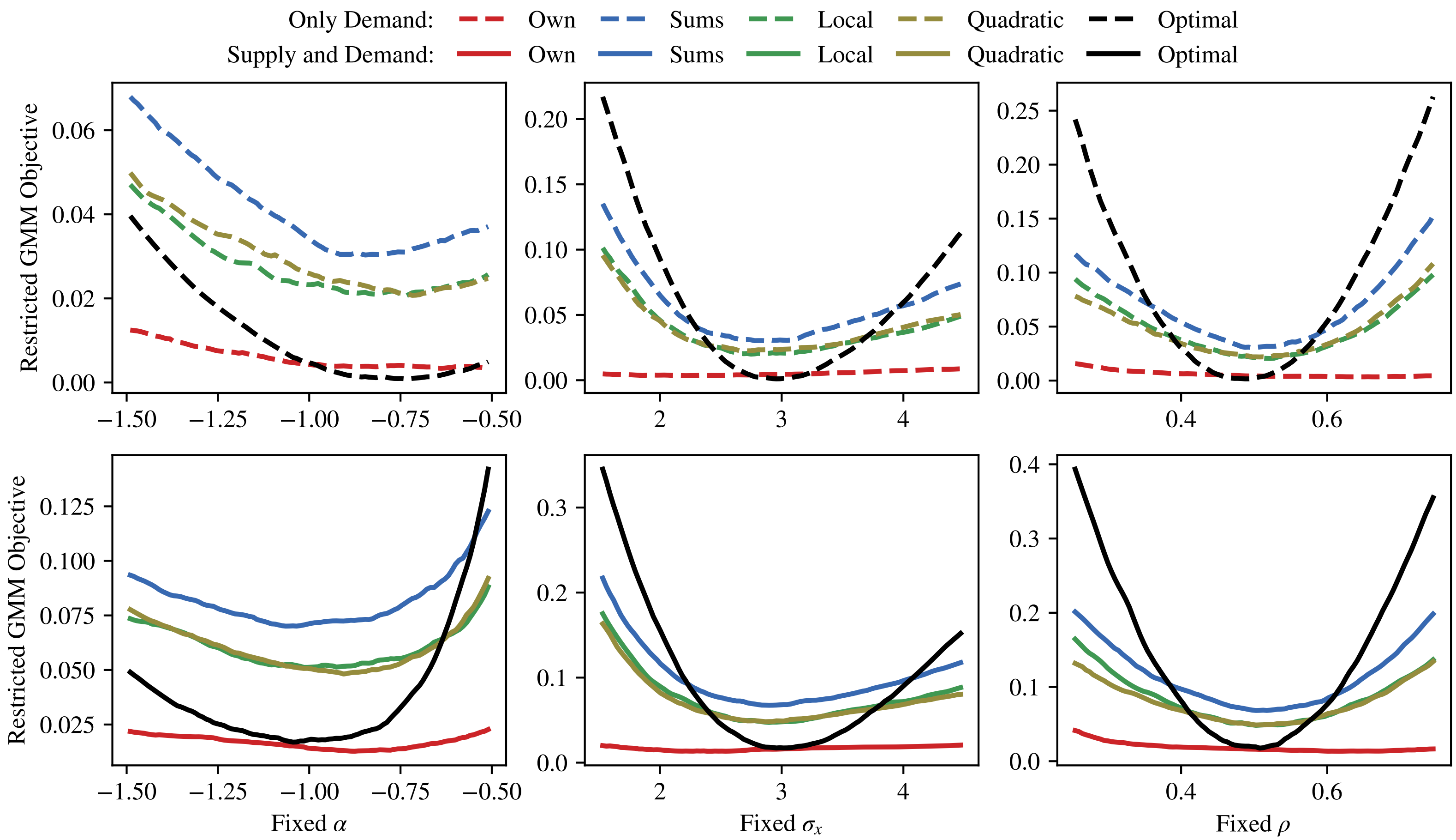
This table documents bias and variance of parameter estimates over the 1,000 simulated datasets with misspecified supply sides used to create the bottom plot in Figure OA11. To increase the strength of the cost shifter w_{jt}, we increase its coefficient γ_w in the equation for marginal costs and report the increasing correlation between w_{jt} and prices p_{jt}. We also report J and LR test statistics to measure goodness of fit.

Figure OA12: Profiled GMM Objective with Alternative Instruments: Complex Simulation



Each plot profiles the GMM objective with respect to a single parameter for our Complex simulation scenario. Otherwise, the plots are the same as those in Figure 3.

Figure OA13: Profiled GMM Objective with Alternative Instruments: RCNL Simulation



Each plot profiles the GMM objective with respect to a single parameter for the RCNL simulation. Otherwise, the plots are the same as those in Figure 3.

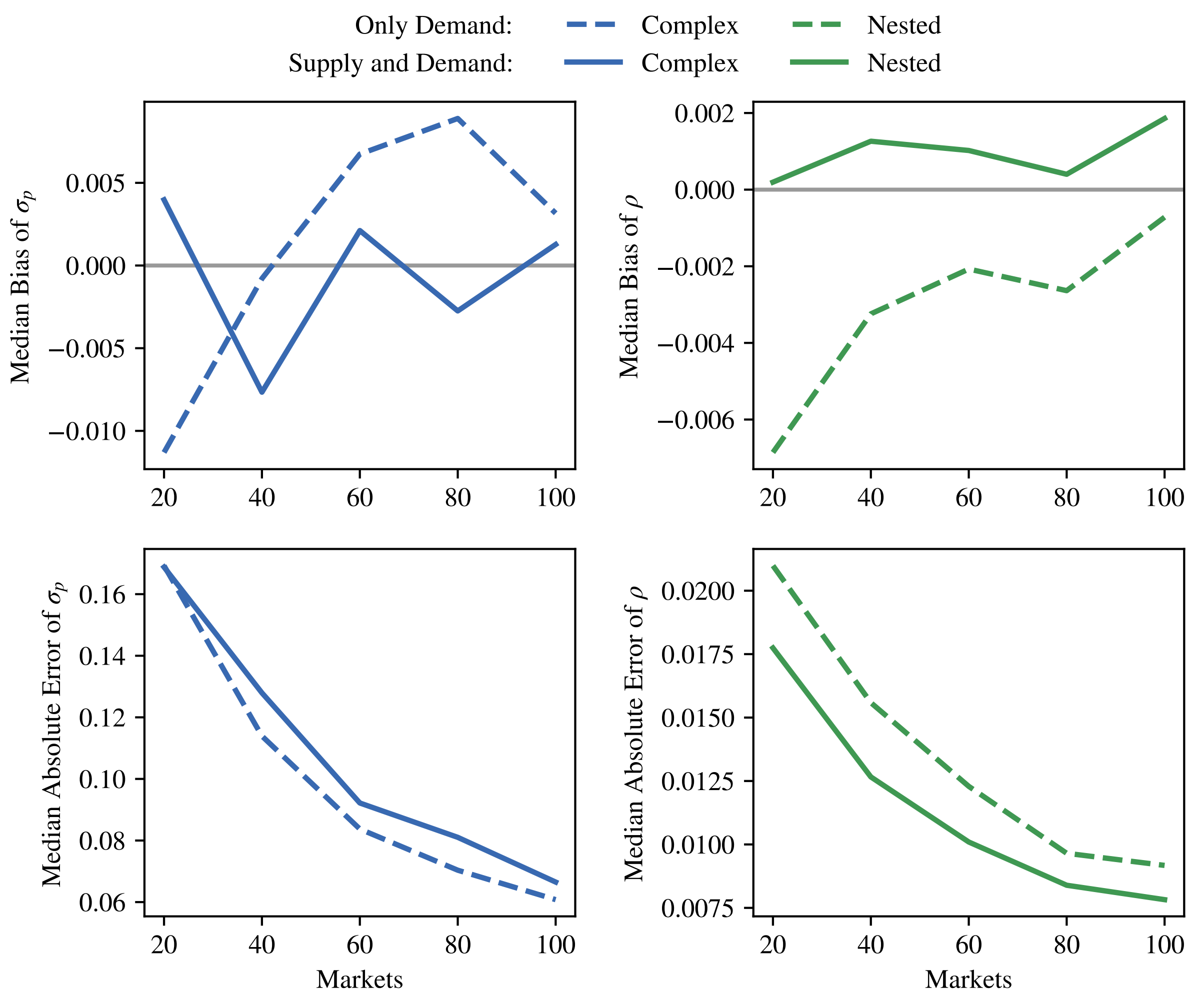
OA4. Problem Scaling

In Figure OA14 we report reductions in bias and variance for parameter estimates not shown in Figure 4 as the number of markets T increases.

In addition to scaling T, we also report results for a larger number of products per firm J_{ft} (Figures OA15 and OA16) and a larger number of firms per market F_t (Figures OA17 and OA18). Although increasing the scale of problems along these other dimensions generally reduces bias and variance, results are more mixed. In particular, our RCNL simulation performs worse as J_{ft} or F_t increases. Intuitively, results are less clean-cut than for scaling T because changing within-market structure changes the characteristics of the problem to be estimated. For example, a larger choice set typically implies a smaller outside good share.

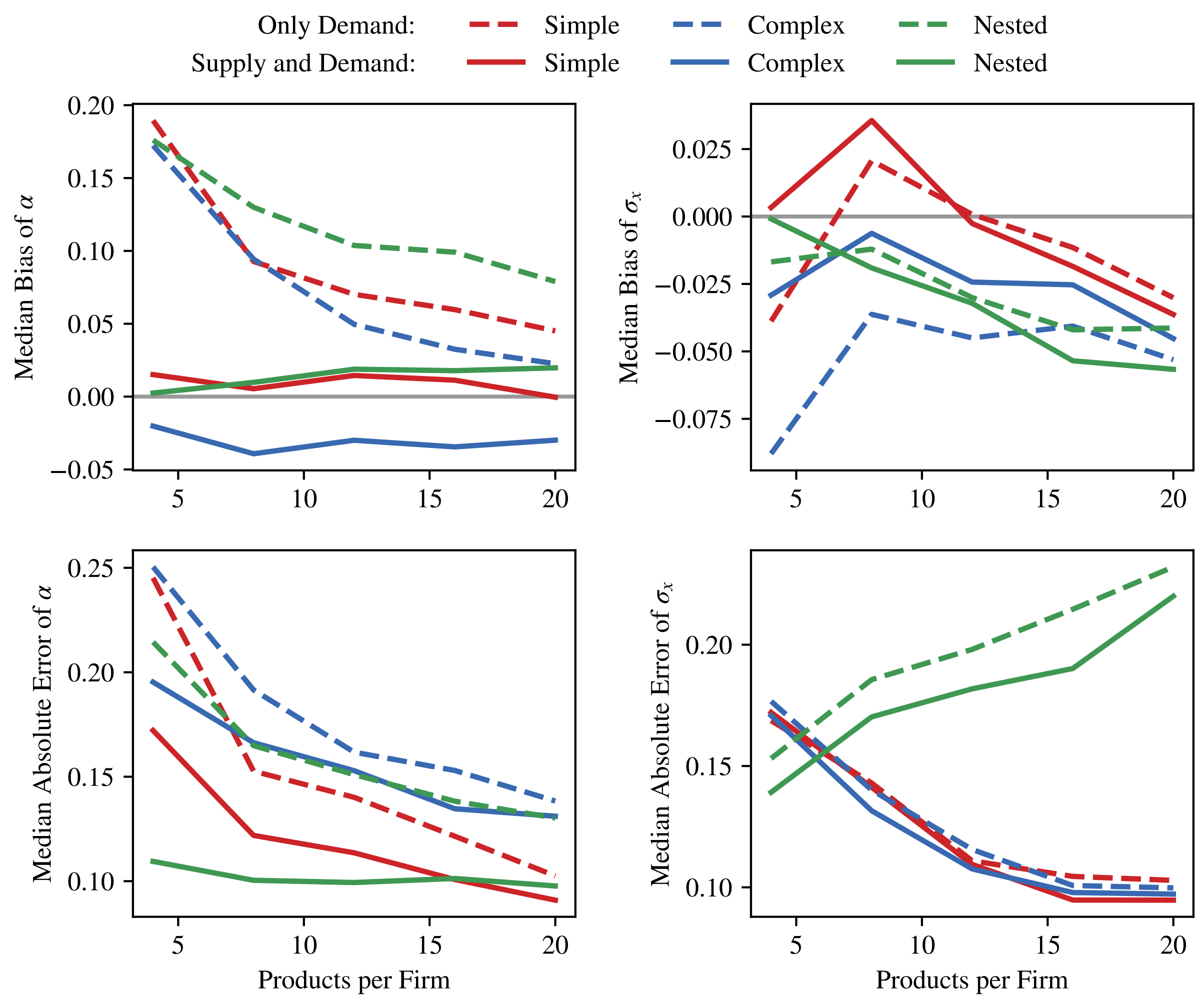
In Table OA6 we summarize all of our problem scaling results in table form. We also document the median number of seconds needed to solve each of the problems, which increases with the problem size. Estimation speed is particularly affected by the number of products per market J_t when a supply side is estimated because the analytic gradient from Appendix A2 involves matrices of shape $J_t \times J_t \times J_t$.

Figure OA14: Problem Scaling: σ_p and ρ



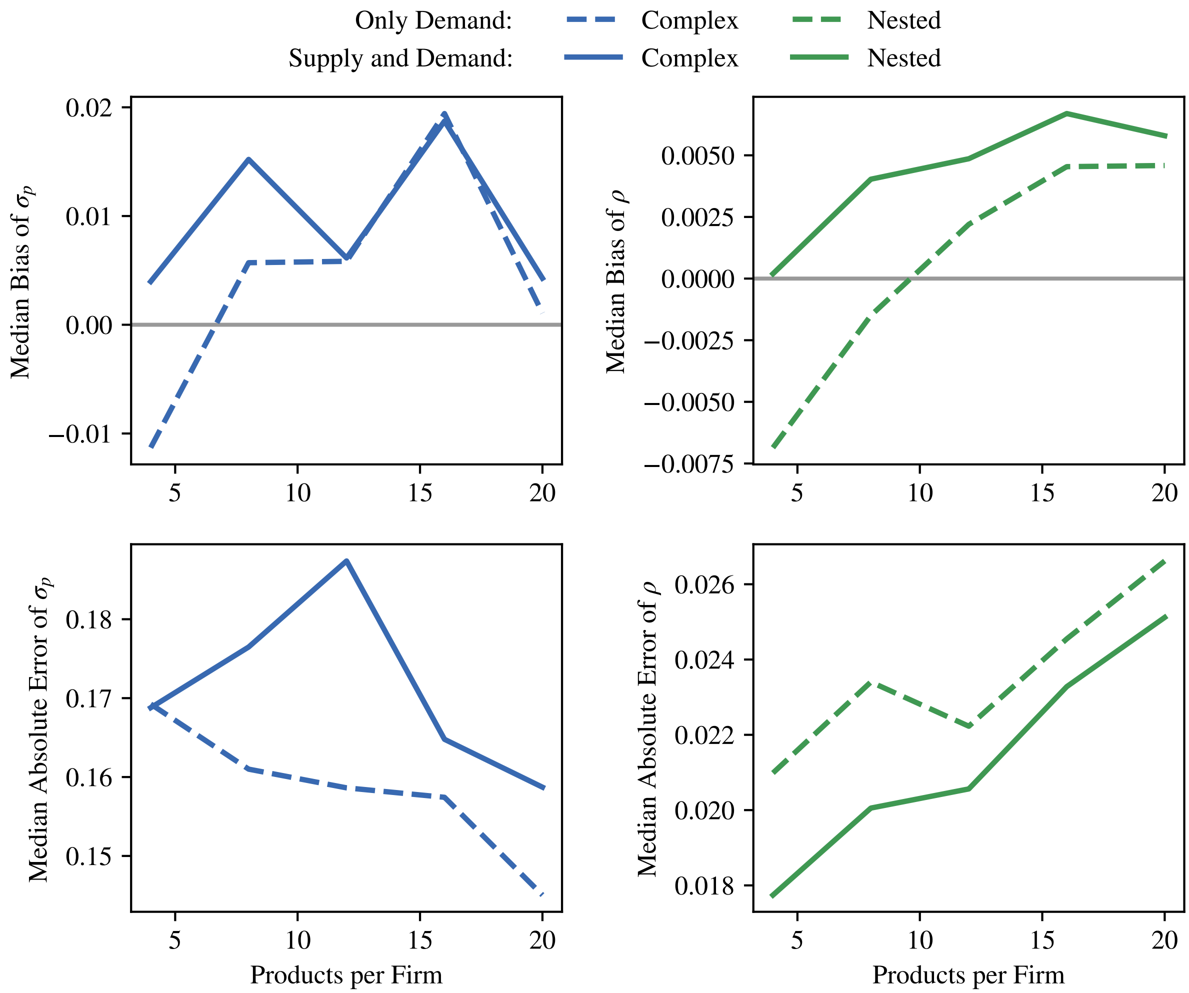
These plots document how bias and variance of σ_p and ρ change with the number of markets T. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.

Figure OA15: Scaling the Number of Products per Firm: α and σ_x



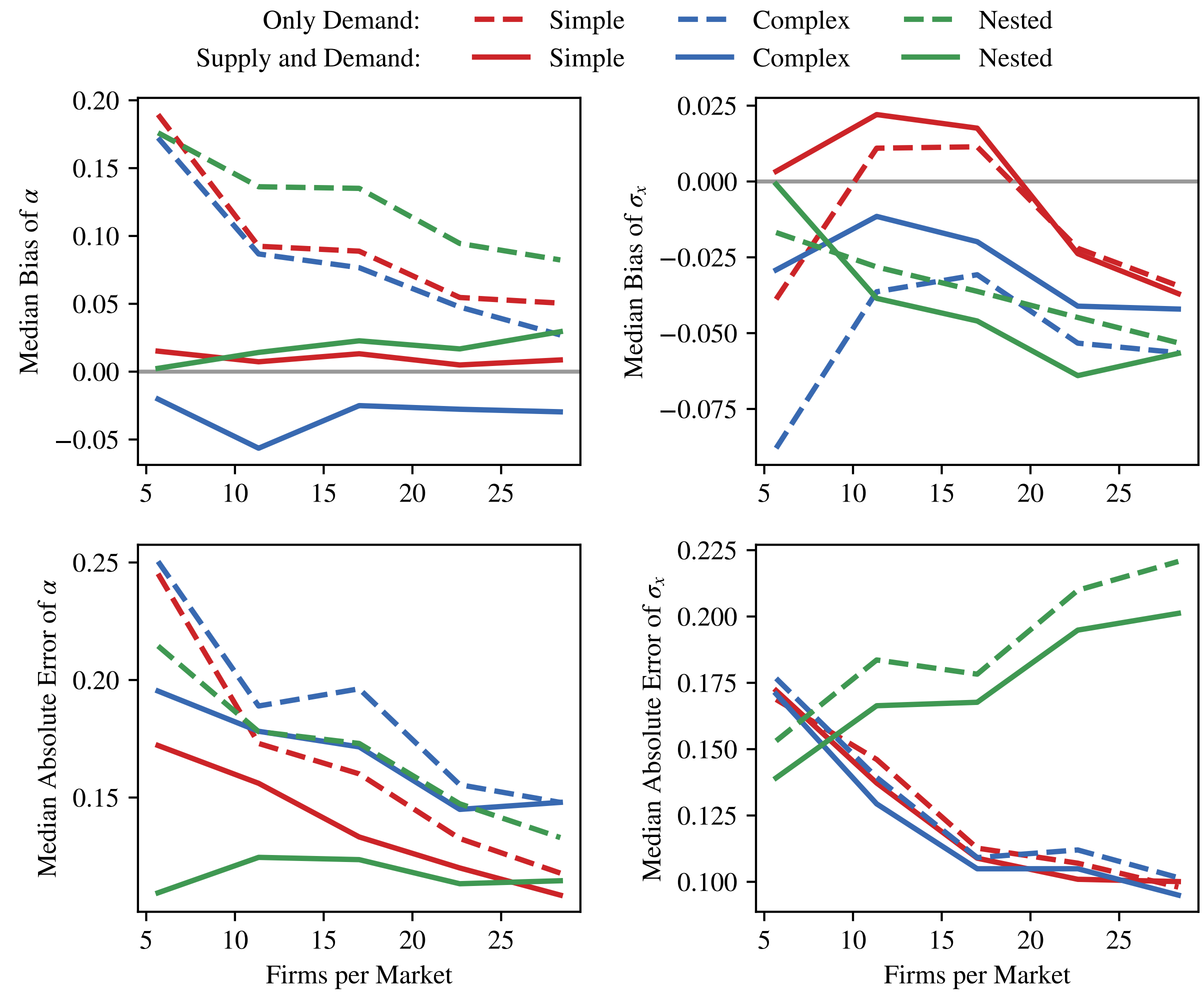
These plots document how bias and variance of α and σ_x change with the mean number of products per firm J_{ft}. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.

Figure OA16: Scaling the Number of Products per Firm: σ_p and ρ



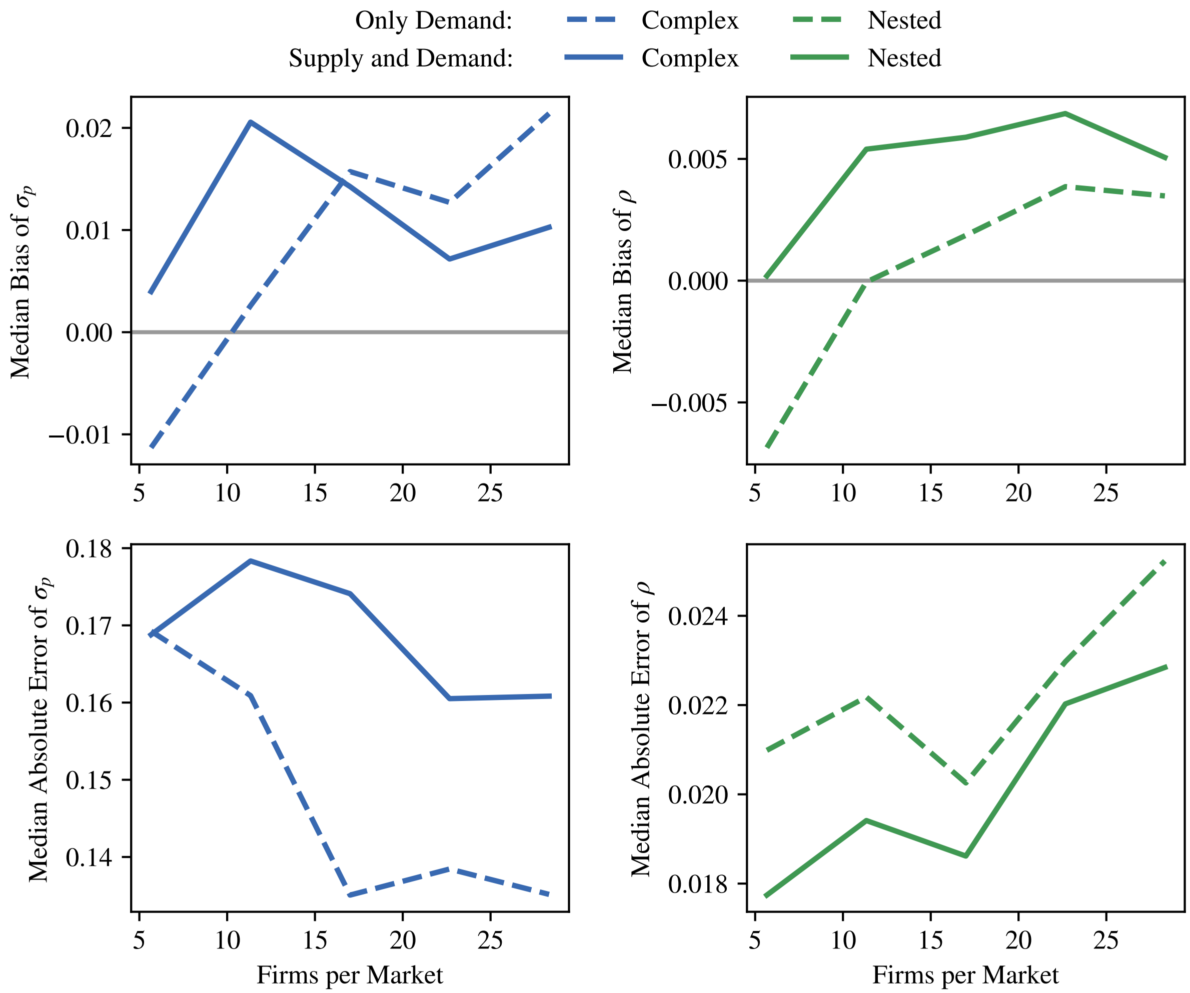
These plots document how bias and variance of σ_p and α change with the mean number of products per firm J_{ft}. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.

Figure OA17: Scaling the Number of Firms per Market: α and σ_x



These plots document how bias and variance of α and σ_x change with the mean number of firms per market F_t. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.

Figure OA18: Scaling the Number of Firms per Market: σ_p and ρ



These plots document how bias and variance of σ_p and α change with the mean number of firms per market F_t. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.

Table OA6: Problem Scaling: Summary

						True Value				Median Bias				Median Absolute Error			
Simulation	Supply	T	J_{ft}	F_t	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	No	20	$\{3, 4, 5\}$	$\{2, 5, 10\}$	0.8	-1	3			0.189	-0.039			0.245	0.169		
Simple	No	100	$\{3, 4, 5\}$	$\{2, 5, 10\}$	3.8	-1	3			0.044	0.018			0.103	0.077		
Simple	No	20	$\{15, 20, 25\}$	$\{2, 5, 10\}$	1.2	-1	3			0.045	-0.030			0.103	0.103		
Simple	No	20	$\{3, 4, 5\}$	$\{10, 25, 50\}$	1.2	-1	3			0.051	-0.034			0.118	0.098		
Simple	Yes	20	$\{3, 4, 5\}$	$\{2, 5, 10\}$	2.2	-1	3			0.015	0.003			0.172	0.172		
Simple	Yes	100	$\{3, 4, 5\}$	$\{2, 5, 10\}$	8.9	-1	3			-0.006	0.020			0.081	0.075		
Simple	Yes	20	$\{15, 20, 25\}$	$\{2, 5, 10\}$	9.6	-1	3			-0.000	-0.036			0.091	0.095		
Simple	Yes	20	$\{3, 4, 5\}$	$\{10, 25, 50\}$	9.5	-1	3			0.009	-0.037			0.108	0.100		
Complex	No	20	$\{3, 4, 5\}$	$\{2, 5, 10\}$	1.6	-1	3	0.2		0.172	-0.088	-0.011		0.250	0.177	0.169	
Complex	No	100	$\{3, 4, 5\}$	$\{2, 5, 10\}$	7.0	-1	3	0.2		0.036	-0.016	0.003		0.106	0.088	0.061	
Complex	No	20	$\{15, 20, 25\}$	$\{2, 5, 10\}$	3.0	-1	3	0.2		0.023	-0.053	0.001		0.138	0.100	0.145	
Complex	No	20	$\{3, 4, 5\}$	$\{10, 25, 50\}$	3.1	-1	3	0.2		0.027	-0.056	0.021		0.148	0.102	0.135	
Complex	Yes	20	$\{3, 4, 5\}$	$\{2, 5, 10\}$	4.7	-1	3	0.2		-0.020	-0.029	0.004		0.195	0.171	0.169	
Complex	Yes	100	$\{3, 4, 5\}$	$\{2, 5, 10\}$	17.2	-1	3	0.2		-0.005	0.003	0.001		0.089	0.077	0.067	
Complex	Yes	20	$\{15, 20, 25\}$	$\{2, 5, 10\}$	26.4	-1	3	0.2		-0.030	-0.045	0.004		0.131	0.097	0.159	
Complex	Yes	20	$\{3, 4, 5\}$	$\{10, 25, 50\}$	25.8	-1	3	0.2		-0.030	-0.042	0.010		0.148	0.095	0.161	
RCNL	No	20	$\{3, 4, 5\}$	$\{2, 5, 10\}$	4.3	-1	3		0.5	0.176	-0.017		-0.007	0.214	0.153		0.021
RCNL	No	100	$\{3, 4, 5\}$	$\{2, 5, 10\}$	18.2	-1	3		0.5	0.045	-0.005		-0.001	0.088	0.074		0.009
RCNL	No	20	$\{15, 20, 25\}$	$\{2, 5, 10\}$	6.0	-1	3		0.5	0.079	-0.041		0.005	0.130	0.232		0.027
RCNL	No	20	$\{3, 4, 5\}$	$\{10, 25, 50\}$	6.0	-1	3		0.5	0.082	-0.053		0.003	0.133	0.221		0.025
RCNL	Yes	20	$\{3, 4, 5\}$	$\{2, 5, 10\}$	9.3	-1	3		0.5	0.002	-0.001		0.000	0.109	0.139		0.018
RCNL	Yes	100	$\{3, 4, 5\}$	$\{2, 5, 10\}$	35.3	-1	3		0.5	-0.001	-0.004		0.002	0.048	0.071		0.008
RCNL	Yes	20	$\{15, 20, 25\}$	$\{2, 5, 10\}$	36.5	-1	3		0.5	0.020	-0.057		0.006	0.098	0.220		0.025
RCNL	Yes	20	$\{3, 4, 5\}$	$\{10, 25, 50\}$	37.3	-1	3		0.5	0.029	-0.057		0.005	0.115	0.201		0.023

This table documents bias and variance of parameter estimates over 1,000 simulated datasets for different problem sizes. We separately increase the number of simulated markets T, the number of products per firm J_{ft}, and the number of firms per market, F_t. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.

OA5. Standard Errors

We report how different estimation practices impact the bias and coverage of standard error estimates. Computed standard errors are the square root of the diagonal of the parameter estimates' covariance matrix:

$$\mathrm{Var}(\hat{\theta}) = (G'WG)^{-1}G'WSWG(G'WG)^{-1} \quad \text{where} \quad S = \frac{1}{N}\sum_{j,t} g_{jt}g'_{jt}.$$

The performance of standard error estimates is more difficult to evaluate than that of point estimates because comparisons are made with respect to a moving target. To compute a parameter's "true" standard error against which we compare its estimated standard error, we compute the standard deviation of the parameter's point estimate across all 1,000 simulations.

In addition to computing the median bias from this "true" value, we also report the fraction of simulations in which a 95% confidence interval covers the true parameter value.

Since these measures of performance are less desirable than our measures of bias and variance of point estimates, they should be interpreted with caution. We report them for completeness' sake.

Table OA7: Standard Errors: Alternative Instruments

				Standard Deviation				Median Bias				Coverage at 0.95			
Simulation	Supply	Instruments	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	No	Own	0.6	0.393	0.456			-0.078	-0.071			0.903	0.958		
Simple	No	Sums	0.6	0.330	1.641			-0.059	-1.120			0.884	0.959		
Simple	No	Local	0.6	0.644	0.969			-0.311	-0.566			0.919	0.957		
Simple	No	Quadratic	0.6	0.466	0.850			-0.119	-0.301			0.929	0.944		
Simple	No	Optimal	0.8	0.303	0.286			-0.040	-0.046			0.862	0.942		
Simple	Yes	Own	1.4	0.399	0.422			-0.113	-0.057			0.891	0.964		
Simple	Yes	Sums	1.5	0.340	1.343			-0.082	-0.847			0.870	0.945		
Simple	Yes	Local	1.4	0.554	0.997			-0.249	-0.614			0.897	0.954		
Simple	Yes	Quadratic	1.5	0.442	0.794			-0.129	-0.271			0.902	0.951		
Simple	Yes	Optimal	2.2	0.337	0.266			-0.106	-0.038			0.908	0.936		
Complex	No	Own	1.1	0.996	0.490	0.555		0.192	0.025	0.649		0.933	0.975	0.624	
Complex	No	Sums	1.5	0.379	1.522	0.220		-0.053	-0.741	-0.058		0.846	0.876	0.397	
Complex	No	Local	1.1	0.607	0.580	0.321		-0.132	-0.101	-0.024		0.907	0.949	0.502	
Complex	No	Quadratic	1.2	0.605	0.686	0.312		-0.141	-0.127	0.009		0.905	0.964	0.561	
Complex	No	Optimal	1.6	0.326	0.256	0.167		-0.064	0.002	-0.037		0.815	0.951	0.602	
Complex	Yes	Own	3.9	0.754	0.438	0.322		-0.042	-0.051	-0.012		0.973	0.956	0.793	
Complex	Yes	Sums	3.7	0.416	1.138	0.191		-0.068	-0.416	-0.047		0.882	0.939	0.681	
Complex	Yes	Local	3.2	0.556	0.440	0.261		-0.075	-0.048	-0.033		0.944	0.941	0.666	
Complex	Yes	Quadratic	3.9	0.524	0.618	0.252		-0.108	-0.147	-0.069		0.921	0.937	0.644	
Complex	Yes	Optimal	4.7	0.358	0.289	0.182		-0.086	-0.045	-0.055		0.911	0.946	0.642	
RCNL	No	Own	5.1	0.500	2.100		0.192	0.034	-0.834		0.067	0.861	0.758		0.998
RCNL	No	Sums	3.1	0.296	1.316		0.061	-0.037	-0.420		-0.011	0.848	0.933		1.000
RCNL	No	Local	3.4	0.344	0.869		0.080	-0.048	-0.411		-0.018	0.891	0.895		1.000
RCNL	No	Quadratic	3.5	0.358	0.885		0.084	-0.057	-0.401		-0.020	0.875	0.874		1.000
RCNL	No	Optimal	4.3	0.253	0.258		0.032	-0.025	-0.020		-0.002	0.851	0.946		1.000
RCNL	Yes	Own	9.7	0.414	1.940		0.195	-0.050	-0.929		-0.008	0.842	0.698		1.000
RCNL	Yes	Sums	6.0	0.341	1.131		0.062	-0.112	-0.325		-0.016	0.872	0.935		0.997
RCNL	Yes	Local	6.8	0.328	0.971		0.083	-0.087	-0.557		-0.026	0.873	0.894		1.000
RCNL	Yes	Quadratic	7.2	0.320	0.936		0.084	-0.084	-0.496		-0.026	0.894	0.878		0.999
RCNL	Yes	Optimal	9.3	0.186	0.239		0.029	-0.040	-0.031		-0.004	0.924	0.934		1.000

This table documents bias and coverage of estimated standard errors over 1,000 simulated datasets for the different instruments compared in Table 5.

Table OA8: Standard Errors: Alternative Integration Methods

					Standard Deviation				Median Bias				Coverage at 0.95			
Simulation	Supply	Integration	I_t	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	No	Monte Carlo	100	1.0	0.360	0.349			-0.080	-0.146			0.797	0.249		
Simple	No	Monte Carlo	1,000	3.1	0.311	0.255			-0.044	-0.031			0.862	0.878		
Simple	No	MLHS	1,000	3.2	0.305	0.253			-0.041	-0.026			0.865	0.938		
Simple	No	Halton	1,000	3.2	0.305	0.253			-0.041	-0.026			0.863	0.936		
Simple	No	Importance	1,000	21.6	0.318	0.284			-0.051	-0.057			0.874	0.917		
Simple	No	Product Rule	9^1	0.8	0.303	0.286			-0.040	-0.046			0.862	0.942		
Simple	Yes	Monte Carlo	100	2.7	0.419	0.367			-0.185	-0.179			0.797	0.212		
Simple	Yes	Monte Carlo	1,000	8.8	0.339	0.252			-0.110	-0.042			0.897	0.856		
Simple	Yes	MLHS	1,000	8.8	0.334	0.248			-0.103	-0.035			0.903	0.926		
Simple	Yes	Halton	1,000	9.2	0.335	0.248			-0.104	-0.035			0.905	0.926		
Simple	Yes	Importance	1,000	27.2	0.346	0.277			-0.114	-0.063			0.915	0.888		
Simple	Yes	Product Rule	9^1	2.2	0.337	0.266			-0.106	-0.038			0.908	0.936		
Complex	No	Monte Carlo	100	1.9	0.358	0.430	0.107		-0.113	-0.222	-0.059		0.694	0.186	0.442	
Complex	No	Monte Carlo	1,000	5.3	0.329	0.275	0.136		-0.073	-0.036	-0.023		0.820	0.864	0.899	
Complex	No	MLHS	1,000	5.1	0.326	0.274	0.148		-0.060	-0.019	0.019		0.850	0.945	0.931	
Complex	No	Halton	1,000	5.7	0.368	0.329	0.170		-0.064	-0.060	0.013		0.894	0.952	0.899	
Complex	No	Importance	1,000	28.3	0.335	0.318	0.141		-0.090	-0.079	-0.052		0.794	0.914	0.783	
Complex	No	Product Rule	9^2	1.6	0.326	0.256	0.167		-0.064	0.002	-0.037		0.815	0.951	0.602	
Complex	Yes	Monte Carlo	100	5.2	0.386	0.516	0.098		-0.172	-0.322	-0.050		0.742	0.227	0.354	
Complex	Yes	Monte Carlo	1,000	15.0	0.367	0.320	0.141		-0.132	-0.102	-0.022		0.855	0.841	0.915	
Complex	Yes	MLHS	1,000	15.2	0.326	0.303	0.154		-0.078	-0.075	0.015		0.878	0.911	0.925	
Complex	Yes	Halton	1,000	17.1	0.382	0.266	0.156		-0.101	-0.031	0.016		0.908	0.935	0.912	
Complex	Yes	Importance	1,000	38.6	0.362	0.326	0.141		-0.136	-0.106	-0.054		0.874	0.869	0.754	
Complex	Yes	Product Rule	9^2	4.7	0.358	0.289	0.182		-0.086	-0.045	-0.055		0.911	0.946	0.642	
RCNL	No	Monte Carlo	100	5.7	0.305	0.351		0.058	-0.077	-0.186		-0.030	0.736	0.190		1.000
RCNL	No	Monte Carlo	1,000	18.9	0.259	0.247		0.033	-0.033	-0.031		-0.004	0.825	0.866		1.000
RCNL	No	MLHS	1,000	19.0	0.253	0.257		0.032	-0.025	-0.026		-0.002	0.853	0.935		1.000
RCNL	No	Halton	1,000	19.9	0.252	0.256		0.032	-0.025	-0.026		-0.002	0.846	0.939		1.000
RCNL	No	Importance	1,000	40.2	0.409	1.328		0.091	-0.121	-0.967		-0.055	0.886	0.447		0.975
RCNL	No	Product Rule	9^1	4.3	0.253	0.258		0.032	-0.025	-0.020		-0.002	0.851	0.946		1.000
RCNL	Yes	Monte Carlo	100	12.5	0.178	0.293		0.049	-0.040	-0.147		-0.026	0.825	0.167		1.000
RCNL	Yes	Monte Carlo	1,000	45.8	0.184	0.223		0.029	-0.041	-0.030		-0.004	0.904	0.876		1.000
RCNL	Yes	MLHS	1,000	45.8	0.187	0.230		0.028	-0.042	-0.027		-0.003	0.921	0.933		1.000
RCNL	Yes	Halton	1,000	47.6	0.187	0.229		0.028	-0.042	-0.025		-0.003	0.920	0.932		1.000
RCNL	Yes	Importance	1,000	66.0	0.265	1.163		0.084	-0.097	-0.857		-0.053	0.923	0.341		0.988
RCNL	Yes	Product Rule	9^1	9.3	0.186	0.239		0.029	-0.040	-0.031		-0.004	0.924	0.934		1.000

This table documents bias and coverage of estimated standard errors over 1,000 simulated datasets for the different numerical integration methods and numbers of integration nodes I_t compared in Table A2.

Table OA9: Standard Errors: Problem Scaling

						Standard Deviation				Median Bias				Coverage at 0.95			
Simulation	Supply	T	J_f	F_t	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	No	20	$\{3,4,5\}$	$\{2,5,10\}$	0.8	0.303	0.286			-0.040	-0.046			0.862	0.942		
Simple	No	100	$\{3,4,5\}$	$\{2,5,10\}$	3.8	0.152	0.120			-0.010	-0.007			0.939	0.935		
Simple	No	20	$\{15,20,25\}$	$\{2,5,10\}$	1.2	0.150	0.147			-0.008	-0.032			0.920	0.876		
Simple	No	20	$\{3,4,5\}$	$\{10,25,50\}$	1.2	0.171	0.143			-0.019	-0.031			0.913	0.875		
Simple	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	2.2	0.337	0.266			-0.106	-0.038			0.908	0.936		
Simple	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	8.9	0.118	0.114			-0.004	-0.008			0.960	0.928		
Simple	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	9.6	0.145	0.142			-0.009	-0.035			0.952	0.854		
Simple	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	9.5	0.172	0.138			-0.016	-0.032			0.944	0.867		
Complex	No	20	$\{3,4,5\}$	$\{2,5,10\}$	1.6	0.326	0.256	0.167		-0.064	0.002	-0.037		0.815	0.951	0.602	
Complex	No	100	$\{3,4,5\}$	$\{2,5,10\}$	7.0	0.163	0.130	0.099		-0.021	-0.001	-0.024		0.911	0.950	0.842	
Complex	No	20	$\{15,20,25\}$	$\{2,5,10\}$	3.0	0.217	0.145	0.174		-0.033	-0.011	-0.036		0.917	0.930	0.741	
Complex	No	20	$\{3,4,5\}$	$\{10,25,50\}$	3.1	0.257	0.168	0.174		-0.065	-0.032	-0.045		0.914	0.925	0.745	
Complex	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	4.7	0.358	0.289	0.182		-0.086	-0.045	-0.055		0.911	0.946	0.642	
Complex	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	17.2	0.139	0.120	0.109		-0.013	-0.005	-0.035		0.915	0.947	0.804	
Complex	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	26.4	0.209	0.140	0.167		-0.022	-0.019	-0.033		0.927	0.907	0.721	
Complex	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	25.8	0.227	0.137	0.167		-0.030	-0.016	-0.045		0.918	0.908	0.678	
RCNL	No	20	$\{3,4,5\}$	$\{2,5,10\}$	4.3	0.253	0.258		0.032	-0.025	-0.020		-0.002	0.851	0.946		1.000
RCNL	No	100	$\{3,4,5\}$	$\{2,5,10\}$	18.2	0.132	0.115		0.014	-0.009	-0.005		-0.000	0.927	0.950		1.000
RCNL	No	20	$\{15,20,25\}$	$\{2,5,10\}$	6.0	0.166	0.394		0.042	-0.017	-0.102		-0.009	0.876	0.916		1.000
RCNL	No	20	$\{3,4,5\}$	$\{10,25,50\}$	6.0	0.175	0.343		0.039	-0.020	-0.064		-0.007	0.868	0.904		1.000
RCNL	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	9.3	0.186	0.239		0.029	-0.040	-0.031		-0.004	0.924	0.934		1.000
RCNL	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	35.3	0.070	0.105		0.012	-0.001	-0.006		-0.001	0.956	0.943		1.000
RCNL	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	36.5	0.151	0.337		0.039	-0.013	-0.065		-0.007	0.927	0.894		1.000
RCNL	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	37.3	0.174	0.313		0.036	-0.018	-0.051		-0.006	0.941	0.905		1.000

This table documents bias and coverage of estimated standard errors over 1,000 simulated datasets for the different problem sizes compared in Table OA6.

OA6. Post-Estimation Outputs

We report how different estimation practices impact the bias and variance of four post-estimation outputs. Mean own-price elasticities are

$$\bar{\varepsilon} = \frac{1}{N} \sum_{j,t} \frac{\partial s_{jt}}{\partial p_{jt}} \cdot \frac{p_{jt}}{s_{jt}}.$$

Mean aggregate price elasticities are

$$\bar{E} = \frac{1}{N} \sum_{j,t} \frac{s_{jt}(p_{jt} + \Delta \cdot p_{jt}) - s_{jt}}{\Delta} \quad \text{where} \quad \Delta = 0.01.$$

Total producer surplus is

$$\text{PS} = \sum_{j,t} (p_{jt} - c_{jt} s_{jt}). \tag{OA1}$$

Total consumer surplus is approximated with

$$\text{CS} \approx \sum_{t,i} w_{it} \cdot \frac{\log(1 + \sum_{j \in J_t} \exp[\delta_{jt} + \mu_{ijt}(\nu_{it})])}{\alpha_i} \quad \text{where} \quad \alpha_i = \frac{\partial u_{ijt}}{\partial p_{jt}}. \tag{OA2}$$

Table OA10: Post-Estimation Outputs: Alternative Instruments

				True Median Value				Median Bias				Median Absolute Error			
Simulation	Supply	Instruments	Seconds	$\bar{\varepsilon}$	$\bar{E}$	PS	CS	$\bar{\varepsilon}$	$\bar{E}$	PS	CS	$\bar{\varepsilon}$	$\bar{E}$	PS	CS
Simple	No	Own	0.6	-3.566	-0.241	2.670	3.688	0.446	0.026	0.319	0.476	0.844	0.054	0.646	0.861
Simple	No	Sums	0.6	-3.566	-0.241	2.670	3.688	0.580	0.036	0.492	0.746	0.868	0.057	0.672	1.006
Simple	No	Local	0.6	-3.566	-0.241	2.670	3.688	0.309	0.019	0.203	0.330	0.866	0.054	0.638	0.872
Simple	No	Quadratic	0.6	-3.566	-0.241	2.670	3.688	0.380	0.023	0.262	0.400	0.913	0.059	0.701	0.985
Simple	No	Optimal	0.8	-3.566	-0.241	2.670	3.688	0.676	0.041	0.575	0.819	0.873	0.055	0.706	0.955
Simple	Yes	Own	1.4	-3.566	-0.241	2.670	3.688	0.083	0.005	0.067	0.142	0.803	0.049	0.576	0.765
Simple	Yes	Sums	1.5	-3.566	-0.241	2.670	3.688	0.142	0.008	0.102	0.222	0.786	0.048	0.577	0.768
Simple	Yes	Local	1.4	-3.566	-0.241	2.670	3.688	0.072	0.003	0.045	0.106	0.817	0.051	0.599	0.785
Simple	Yes	Quadratic	1.5	-3.566	-0.241	2.670	3.688	0.024	0.006	0.030	0.169	0.882	0.054	0.629	0.870
Simple	Yes	Optimal	2.2	-3.566	-0.241	2.670	3.688	0.055	0.001	0.041	0.068	0.611	0.039	0.445	0.583
Complex	No	Own	1.1	-3.305	-0.227	2.958	4.543	0.399	0.024	0.223	0.319	0.903	0.050	0.772	1.589
Complex	No	Sums	1.5	-3.305	-0.227	2.958	4.543	0.739	0.039	0.829	1.099	0.871	0.053	0.945	1.701
Complex	No	Local	1.1	-3.305	-0.227	2.958	4.543	0.541	0.031	0.518	0.631	0.882	0.052	0.826	1.440
Complex	No	Quadratic	1.2	-3.305	-0.227	2.958	4.543	0.618	0.038	0.515	0.775	0.905	0.053	0.839	1.584
Complex	No	Optimal	1.6	-3.305	-0.227	2.958	4.543	0.723	0.044	0.788	1.122	0.868	0.052	0.860	1.494
Complex	Yes	Own	3.9	-3.305	-0.227	2.958	4.543	0.226	0.017	0.076	0.767	0.935	0.048	0.816	2.223
Complex	Yes	Sums	3.7	-3.305	-0.227	2.958	4.543	0.262	0.006	0.188	0.461	0.741	0.046	0.633	1.364
Complex	Yes	Local	3.2	-3.305	-0.227	2.958	4.543	0.215	0.009	0.133	0.516	0.764	0.043	0.646	1.447
Complex	Yes	Quadratic	3.9	-3.305	-0.227	2.958	4.543	0.466	0.030	0.314	0.985	0.987	0.056	0.816	2.303
Complex	Yes	Optimal	4.7	-3.305	-0.227	2.958	4.543	0.048	0.001	0.032	0.084	0.575	0.034	0.486	0.898
RCNL	No	Own	5.1	-5.893	-0.194	1.634	3.212	0.502	0.052	0.153	1.069	2.152	0.063	0.631	1.457
RCNL	No	Sums	3.1	-5.893	-0.194	1.634	3.212	0.924	0.022	0.280	0.498	1.304	0.039	0.381	0.756
RCNL	No	Local	3.4	-5.893	-0.194	1.634	3.212	0.612	0.021	0.196	0.460	1.263	0.039	0.367	0.684
RCNL	No	Quadratic	3.5	-5.893	-0.194	1.634	3.212	0.678	0.021	0.212	0.468	1.347	0.041	0.388	0.723
RCNL	No	Optimal	4.3	-5.893	-0.194	1.634	3.212	1.110	0.032	0.367	0.681	1.321	0.039	0.410	0.769
RCNL	Yes	Own	9.7	-5.893	-0.194	1.634	3.212	-1.235	0.011	-0.138	0.388	1.757	0.038	0.357	0.710
RCNL	Yes	Sums	6.0	-5.893	-0.194	1.634	3.212	0.099	-0.006	0.010	-0.004	1.092	0.035	0.291	0.589
RCNL	Yes	Local	6.8	-5.893	-0.194	1.634	3.212	-0.031	0.002	0.011	0.102	1.110	0.033	0.286	0.546
RCNL	Yes	Quadratic	7.2	-5.893	-0.194	1.634	3.212	-0.070	0.001	-0.003	0.103	1.090	0.033	0.288	0.539
RCNL	Yes	Optimal	9.3	-5.893	-0.194	1.634	3.212	0.022	-0.000	0.008	0.042	0.647	0.020	0.176	0.335

This table documents bias and variance of post-estimation outputs over 1,000 simulated datasets for the different instruments compared in Table 5.

Table OA11: Post-Estimation Outputs: Alternative Integration Methods

					True Median Value				Median Bias				Median Absolute Error			
Simulation	Supply	Integration	I_t	Seconds	$\bar{\varepsilon}$	$\bar{E}$	PS	CS	$\bar{\varepsilon}$	$\bar{E}$	PS	CS	$\bar{\varepsilon}$	$\bar{E}$	PS	CS
Simple	No	Monte Carlo	100	1.0	-3.566	-0.241	2.670	3.688	0.812	0.024	0.583	0.367	1.046	0.066	0.826	1.044
Simple	No	Monte Carlo	1,000	3.1	-3.566	-0.241	2.670	3.688	0.703	0.040	0.583	0.741	0.888	0.055	0.711	0.919
Simple	No	MLHS	1,000	3.2	-3.566	-0.241	2.670	3.688	0.666	0.042	0.576	0.784	0.858	0.054	0.702	0.947
Simple	No	Halton	1,000	3.2	-3.566	-0.241	2.670	3.688	0.665	0.042	0.575	0.762	0.858	0.055	0.700	0.947
Simple	No	Importance	1,000	21.6	-3.566	-0.241	2.670	3.688	0.648	0.046	0.571	0.835	0.861	0.057	0.697	0.991
Simple	No	Product Rule	9^1	0.8	-3.566	-0.241	2.670	3.688	0.676	0.041	0.575	0.819	0.873	0.055	0.706	0.955
Simple	Yes	Monte Carlo	100	2.7	-3.566	-0.241	2.670	3.688	0.384	-0.007	0.233	-0.065	0.853	0.052	0.626	0.802
Simple	Yes	Monte Carlo	1,000	8.8	-3.566	-0.241	2.670	3.688	0.073	0.001	0.044	0.028	0.639	0.040	0.470	0.608
Simple	Yes	MLHS	1,000	8.8	-3.566	-0.241	2.670	3.688	0.071	0.004	0.047	0.055	0.613	0.038	0.447	0.595
Simple	Yes	Halton	1,000	9.2	-3.566	-0.241	2.670	3.688	0.071	0.004	0.051	0.047	0.610	0.038	0.443	0.592
Simple	Yes	Importance	1,000	27.2	-3.566	-0.241	2.670	3.688	-0.004	0.003	0.006	0.044	0.625	0.038	0.436	0.592
Simple	Yes	Product Rule	9^1	2.2	-3.566	-0.241	2.670	3.688	0.055	0.001	0.041	0.068	0.611	0.039	0.445	0.583
Complex	No	Monte Carlo	100	1.9	-3.305	-0.227	2.958	4.543	0.958	0.036	1.021	0.581	1.045	0.063	1.157	1.500
Complex	No	Monte Carlo	1,000	5.3	-3.305	-0.227	2.958	4.543	0.771	0.043	0.799	1.036	0.916	0.053	0.896	1.545
Complex	No	MLHS	1,000	5.1	-3.305	-0.227	2.958	4.543	0.722	0.045	0.779	1.200	0.875	0.053	0.865	1.503
Complex	No	Halton	1,000	5.7	-3.305	-0.227	2.958	4.543	0.739	0.042	0.747	1.261	0.887	0.051	0.844	1.633
Complex	No	Importance	1,000	28.3	-3.305	-0.227	2.958	4.543	0.738	0.045	0.768	1.220	0.891	0.054	0.862	1.576
Complex	No	Product Rule	9^2	1.6	-3.305	-0.227	2.958	4.543	0.723	0.044	0.788	1.122	0.868	0.052	0.860	1.494
Complex	Yes	Monte Carlo	100	5.2	-3.305	-0.227	2.958	4.543	0.165	-0.019	0.098	-0.538	0.763	0.053	0.660	1.033
Complex	Yes	Monte Carlo	1,000	15.0	-3.305	-0.227	2.958	4.543	0.091	0.001	0.064	0.014	0.581	0.036	0.512	0.820
Complex	Yes	MLHS	1,000	15.2	-3.305	-0.227	2.958	4.543	0.072	0.003	0.060	0.086	0.565	0.035	0.490	0.831
Complex	Yes	Halton	1,000	17.1	-3.305	-0.227	2.958	4.543	0.077	0.002	0.038	0.179	0.573	0.034	0.479	0.877
Complex	Yes	Importance	1,000	38.6	-3.305	-0.227	2.958	4.543	-0.126	-0.004	-0.096	-0.085	0.563	0.035	0.480	0.802
Complex	Yes	Product Rule	9^2	4.7	-3.305	-0.227	2.958	4.543	0.048	0.001	0.032	0.084	0.575	0.034	0.486	0.898
RCNL	No	Monte Carlo	100	5.7	-5.893	-0.194	1.634	3.212	0.876	0.021	0.280	0.412	1.474	0.043	0.413	0.744
RCNL	No	Monte Carlo	1,000	18.9	-5.893	-0.194	1.634	3.212	1.068	0.030	0.335	0.582	1.304	0.038	0.389	0.683
RCNL	No	MLHS	1,000	19.0	-5.893	-0.194	1.634	3.212	1.114	0.032	0.352	0.643	1.338	0.040	0.405	0.755
RCNL	No	Halton	1,000	19.9	-5.893	-0.194	1.634	3.212	1.105	0.032	0.351	0.629	1.339	0.040	0.411	0.749
RCNL	No	Importance	1,000	40.2	-5.893	-0.194	1.634	3.212	1.399	0.048	0.419	1.004	1.642	0.051	0.492	1.070
RCNL	No	Product Rule	9^1	4.3	-5.893	-0.194	1.634	3.212	1.110	0.032	0.367	0.681	1.321	0.039	0.410	0.769
RCNL	Yes	Monte Carlo	100	12.5	-5.893	-0.194	1.634	3.212	-0.156	-0.011	-0.027	-0.131	0.753	0.023	0.185	0.348
RCNL	Yes	Monte Carlo	1,000	45.8	-5.893	-0.194	1.634	3.212	0.032	0.001	0.011	0.013	0.688	0.020	0.183	0.337
RCNL	Yes	MLHS	1,000	45.8	-5.893	-0.194	1.634	3.212	0.038	0.002	0.013	0.035	0.661	0.020	0.180	0.333
RCNL	Yes	Halton	1,000	47.6	-5.893	-0.194	1.634	3.212	0.032	0.002	0.015	0.031	0.674	0.020	0.179	0.332
RCNL	Yes	Importance	1,000	66.0	-5.893	-0.194	1.634	3.212	0.644	0.024	0.154	0.439	0.875	0.030	0.233	0.512
RCNL	Yes	Product Rule	9^1	9.3	-5.893	-0.194	1.634	3.212	0.022	-0.000	0.008	0.042	0.647	0.020	0.176	0.335

This table documents bias and variance of post-estimation outputs over 1,000 simulated datasets for the different numerical integration methods and numbers of integration nodes I_t compared in Table A2.

Table OA12: Post-Estimation Outputs: Problem Scaling

						True Median Value				Median Bias				Median Absolute Error			
Simulation	Supply	T	J_f	F_t	Seconds	$\bar{\varepsilon}$	$\bar{E}$	PS	CS	$\bar{\varepsilon}$	$\bar{E}$	PS	CS	$\bar{\varepsilon}$	$\bar{E}$	PS	CS
Simple	No	20	$\{3,4,5\}$	$\{2,5,10\}$	0.8	-3.566	-0.241	2.670	3.688	0.676	0.041	0.575	0.819	0.873	0.055	0.706	0.955
Simple	No	100	$\{3,4,5\}$	$\{2,5,10\}$	3.8	-3.565	-0.242	13.317	18.560	0.157	0.009	0.614	0.960	0.368	0.023	1.387	1.856
Simple	No	20	$\{15,20,25\}$	$\{2,5,10\}$	1.2	-3.633	-0.410	5.664	9.882	0.164	0.020	0.277	0.342	0.373	0.044	0.593	1.025
Simple	No	20	$\{3,4,5\}$	$\{10,25,50\}$	1.2	-3.591	-0.407	5.454	10.131	0.181	0.022	0.284	0.380	0.423	0.048	0.659	1.177
Simple	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	2.2	-3.566	-0.241	2.670	3.688	0.055	0.001	0.041	0.068	0.611	0.039	0.445	0.583
Simple	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	8.9	-3.565	-0.242	13.317	18.560	-0.019	-0.002	-0.061	0.098	0.289	0.018	1.025	1.351
Simple	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	9.6	-3.633	-0.410	5.664	9.882	-0.002	0.003	-0.006	-0.094	0.332	0.038	0.524	0.905
Simple	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	9.5	-3.591	-0.407	5.454	10.131	0.031	0.006	0.050	-0.050	0.390	0.045	0.583	1.104
Complex	No	20	$\{3,4,5\}$	$\{2,5,10\}$	1.6	-3.305	-0.227	2.958	4.543	0.723	0.044	0.788	1.122	0.868	0.052	0.860	1.494
Complex	No	100	$\{3,4,5\}$	$\{2,5,10\}$	7.0	-3.303	-0.228	14.767	22.782	0.187	0.011	0.839	1.214	0.365	0.022	1.627	2.880
Complex	No	20	$\{15,20,25\}$	$\{2,5,10\}$	3.0	-3.415	-0.389	6.040	11.680	0.197	0.017	0.291	0.825	0.381	0.041	0.696	1.789
Complex	No	20	$\{3,4,5\}$	$\{10,25,50\}$	3.1	-3.377	-0.387	5.818	11.987	0.246	0.021	0.360	1.164	0.440	0.046	0.747	2.127
Complex	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	4.7	-3.305	-0.227	2.958	4.543	0.048	0.001	0.032	0.084	0.575	0.034	0.486	0.898
Complex	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	17.2	-3.303	-0.228	14.767	22.782	-0.020	-0.002	-0.069	0.166	0.248	0.015	1.040	2.010
Complex	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	26.4	-3.415	-0.389	6.040	11.680	-0.022	-0.006	-0.095	-0.005	0.300	0.034	0.551	1.559
Complex	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	25.8	-3.377	-0.387	5.818	11.987	0.008	-0.003	-0.049	0.050	0.374	0.041	0.626	1.721
RCNL	No	20	$\{3,4,5\}$	$\{2,5,10\}$	4.3	-5.893	-0.194	1.634	3.212	1.110	0.032	0.367	0.681	1.321	0.039	0.410	0.769
RCNL	No	100	$\{3,4,5\}$	$\{2,5,10\}$	18.2	-5.892	-0.194	8.204	16.095	0.279	0.007	0.406	0.816	0.561	0.016	0.784	1.435
RCNL	No	20	$\{15,20,25\}$	$\{2,5,10\}$	6.0	-6.203	-0.274	2.247	6.495	0.429	0.012	0.166	0.457	0.721	0.028	0.270	0.755
RCNL	No	20	$\{3,4,5\}$	$\{10,25,50\}$	6.0	-6.131	-0.273	2.135	6.630	0.454	0.014	0.173	0.491	0.756	0.031	0.268	0.813
RCNL	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	9.3	-5.893	-0.194	1.634	3.212	0.022	-0.000	0.008	0.042	0.647	0.020	0.176	0.335
RCNL	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	35.3	-5.892	-0.194	8.204	16.095	-0.035	-0.002	-0.027	0.056	0.297	0.009	0.390	0.700
RCNL	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	36.5	-6.203	-0.274	2.247	6.495	0.035	-0.006	0.014	0.020	0.563	0.024	0.206	0.584
RCNL	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	37.3	-6.131	-0.273	2.135	6.630	0.097	-0.003	0.032	0.077	0.652	0.028	0.229	0.682

This table documents bias and variance of post-estimation outputs over 1,000 simulated datasets for the different problem sizes compared in Table OA6.

OA7. Merger Simulation

We report how different estimation practices impact the bias and variance of three merger simulation outputs. After a merger of three firms, we compute post-merger prices and shares with the ζ-markup approach in Section 3.6. We then compute the post-merger change in total producer surplus, PS from (OA1), the change in total consumer surplus, CS from (OA2), and the change in the mean Herfindahl-Hirschman Index,

$$\overline{\mathrm{HHI}} = \frac{10{,}000}{T} \sum_{t,f} \Big(\sum_{j \in J_{ft}} s_{jt} \Big)^2.$$

Table OA13: Merger Simulation: Alternative Instruments

				True Median Value			Median Bias			Median Absolute Error		
Simulation	Supply	Instruments	Seconds	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS
Simple	No	Own	0.6	1,114.102	0.051	-0.168	2.209	0.004	-0.020	12.177	0.013	0.041
Simple	No	Sums	0.6	1,114.102	0.051	-0.168	1.655	0.007	-0.030	19.578	0.016	0.048
Simple	No	Local	0.6	1,114.102	0.051	-0.168	1.703	0.002	-0.012	13.925	0.013	0.040
Simple	No	Quadratic	0.6	1,114.102	0.051	-0.168	1.667	0.003	-0.016	19.088	0.015	0.046
Simple	No	Optimal	0.8	1,114.102	0.051	-0.168	1.690	0.009	-0.036	8.134	0.012	0.044
Simple	Yes	Own	1.4	1,114.102	0.051	-0.168	0.040	0.000	-0.006	11.548	0.012	0.037
Simple	Yes	Sums	1.5	1,114.102	0.051	-0.168	1.439	-0.000	-0.007	19.851	0.014	0.038
Simple	Yes	Local	1.4	1,114.102	0.051	-0.168	0.877	0.000	-0.005	13.501	0.012	0.038
Simple	Yes	Quadratic	1.5	1,114.102	0.051	-0.168	0.525	0.000	-0.004	18.291	0.014	0.043
Simple	Yes	Optimal	2.2	1,114.102	0.051	-0.168	0.252	-0.001	-0.002	8.493	0.008	0.027
Complex	No	Own	1.1	1,106.325	0.060	-0.197	20.913	-0.009	0.022	28.889	0.025	0.082
Complex	No	Sums	1.5	1,106.325	0.060	-0.197	15.007	0.001	-0.016	27.997	0.028	0.074
Complex	No	Local	1.1	1,106.325	0.060	-0.197	11.792	-0.001	-0.003	23.795	0.019	0.065
Complex	No	Quadratic	1.2	1,106.325	0.060	-0.197	16.872	-0.002	0.001	32.646	0.024	0.073
Complex	No	Optimal	1.6	1,106.325	0.060	-0.197	4.374	0.007	-0.033	15.943	0.016	0.057
Complex	Yes	Own	3.9	1,106.325	0.060	-0.197	41.301	-0.044	0.140	118.431	26.108	22.743
Complex	Yes	Sums	3.7	1,106.325	0.060	-0.197	6.278	-0.005	-0.003	20.833	0.017	0.045
Complex	Yes	Local	3.2	1,106.325	0.060	-0.197	-0.004	-0.003	0.009	28.884	0.016	0.057
Complex	Yes	Quadratic	3.9	1,106.325	0.060	-0.197	31.174	-0.021	0.059	67.206	0.041	0.141
Complex	Yes	Optimal	4.7	1,106.325	0.060	-0.197	-1.640	0.001	-0.004	12.636	0.010	0.033
RCNL	No	Own	5.1	926.219	0.069	-0.157	-11.958	0.022	-0.036	18.922	0.033	0.066
RCNL	No	Sums	3.1	926.219	0.069	-0.157	8.065	0.006	-0.020	23.038	0.018	0.036
RCNL	No	Local	3.4	926.219	0.069	-0.157	-0.899	0.009	-0.021	7.593	0.015	0.035
RCNL	No	Quadratic	3.5	926.219	0.069	-0.157	-0.913	0.009	-0.021	7.977	0.015	0.037
RCNL	No	Optimal	4.3	926.219	0.069	-0.157	0.213	0.014	-0.034	4.817	0.016	0.039
RCNL	Yes	Own	9.7	926.219	0.069	-0.157	-12.297	0.006	-0.002	15.728	0.015	0.032
RCNL	Yes	Sums	6.0	926.219	0.069	-0.157	6.174	-0.003	0.002	22.536	0.017	0.032
RCNL	Yes	Local	6.8	926.219	0.069	-0.157	-2.253	0.002	-0.005	6.916	0.013	0.028
RCNL	Yes	Quadratic	7.2	926.219	0.069	-0.157	-2.443	0.001	-0.004	6.952	0.012	0.027
RCNL	Yes	Optimal	9.3	926.219	0.069	-0.157	-1.574	0.001	-0.003	4.489	0.007	0.017

This table documents bias and variance of merger simulation outputs over 1,000 simulated datasets the for different instruments considered in Table 5.

Table OA14: Merger Simulation: Alternative Integration Methods

					True Median Value			Median Bias			Median Absolute Error		
Simulation	Supply	Integration	I_t	Seconds	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS
Simple	No	Monte Carlo	100	1.0	1,114.102	0.051	-0.168	36.612	-0.011	0.007	36.612	0.017	0.046
Simple	No	Monte Carlo	1,000	3.1	1,114.102	0.051	-0.168	6.709	0.006	-0.027	9.888	0.012	0.041
Simple	No	MLHS	1,000	3.2	1,114.102	0.051	-0.168	2.318	0.009	-0.033	8.006	0.013	0.044
Simple	No	Halton	1,000	3.2	1,114.102	0.051	-0.168	2.182	0.009	-0.033	8.062	0.013	0.044
Simple	No	Importance	1,000	21.6	1,114.102	0.051	-0.168	-0.979	0.011	-0.037	8.281	0.014	0.046
Simple	No	Product Rule	9^1	0.8	1,114.102	0.051	-0.168	1.690	0.009	-0.036	8.134	0.012	0.044
Simple	Yes	Monte Carlo	100	2.7	1,114.102	0.051	-0.168	37.792	-0.016	0.024	37.893	0.016	0.039
Simple	Yes	Monte Carlo	1,000	8.8	1,114.102	0.051	-0.168	5.001	-0.002	0.001	9.477	0.009	0.028
Simple	Yes	MLHS	1,000	8.8	1,114.102	0.051	-0.168	0.604	0.000	-0.002	8.281	0.008	0.027
Simple	Yes	Halton	1,000	9.2	1,114.102	0.051	-0.168	0.480	0.000	-0.002	8.282	0.008	0.027
Simple	Yes	Importance	1,000	27.2	1,114.102	0.051	-0.168	-2.997	0.002	-0.003	9.276	0.009	0.027
Simple	Yes	Product Rule	9^1	2.2	1,114.102	0.051	-0.168	0.252	-0.001	-0.002	8.493	0.008	0.027
Complex	No	Monte Carlo	100	1.9	1,106.325	0.060	-0.197	50.686	-0.017	0.016	50.713	0.023	0.063
Complex	No	Monte Carlo	1,000	5.3	1,106.325	0.060	-0.197	8.482	0.004	-0.023	15.684	0.016	0.059
Complex	No	MLHS	1,000	5.1	1,106.325	0.060	-0.197	4.218	0.007	-0.029	13.746	0.017	0.060
Complex	No	Halton	1,000	5.7	1,106.325	0.060	-0.197	3.468	0.005	-0.022	15.974	0.018	0.064
Complex	No	Importance	1,000	28.3	1,106.325	0.060	-0.197	0.774	0.009	-0.032	12.829	0.018	0.063
Complex	No	Product Rule	9^2	1.6	1,106.325	0.060	-0.197	4.374	0.007	-0.033	15.943	0.016	0.057
Complex	Yes	Monte Carlo	100	5.2	1,106.325	0.060	-0.197	45.452	-0.021	0.041	45.784	0.022	0.050
Complex	Yes	Monte Carlo	1,000	15.0	1,106.325	0.060	-0.197	6.475	-0.003	0.004	12.243	0.010	0.032
Complex	Yes	MLHS	1,000	15.2	1,106.325	0.060	-0.197	2.573	-0.001	-0.001	11.702	0.010	0.032
Complex	Yes	Halton	1,000	17.1	1,106.325	0.060	-0.197	-1.339	0.000	-0.005	12.418	0.009	0.032
Complex	Yes	Importance	1,000	38.6	1,106.325	0.060	-0.197	-3.460	0.000	0.003	10.425	0.010	0.032
Complex	Yes	Product Rule	9^2	4.7	1,106.325	0.060	-0.197	-1.640	0.001	-0.004	12.636	0.010	0.033
RCNL	No	Monte Carlo	100	5.7	926.219	0.069	-0.157	14.989	0.005	-0.016	15.185	0.016	0.038
RCNL	No	Monte Carlo	1,000	18.9	926.219	0.069	-0.157	3.781	0.011	-0.029	5.638	0.015	0.035
RCNL	No	MLHS	1,000	19.0	926.219	0.069	-0.157	1.903	0.013	-0.031	4.791	0.016	0.038
RCNL	No	Halton	1,000	19.9	926.219	0.069	-0.157	2.020	0.013	-0.032	4.760	0.016	0.038
RCNL	No	Importance	1,000	40.2	926.219	0.069	-0.157	-9.489	0.025	-0.047	14.744	0.026	0.051
RCNL	No	Product Rule	9^1	4.3	926.219	0.069	-0.157	0.213	0.014	-0.034	4.817	0.016	0.039
RCNL	Yes	Monte Carlo	100	12.5	926.219	0.069	-0.157	12.732	-0.006	0.009	13.131	0.009	0.018
RCNL	Yes	Monte Carlo	1,000	45.8	926.219	0.069	-0.157	1.455	-0.000	0.000	4.420	0.007	0.016
RCNL	Yes	MLHS	1,000	45.8	926.219	0.069	-0.157	-0.008	0.000	-0.001	3.843	0.007	0.016
RCNL	Yes	Halton	1,000	47.6	926.219	0.069	-0.157	0.060	0.000	-0.001	3.890	0.007	0.016
RCNL	Yes	Importance	1,000	66.0	926.219	0.069	-0.157	-12.243	0.012	-0.021	14.903	0.013	0.027
RCNL	Yes	Product Rule	9^1	9.3	926.219	0.069	-0.157	-1.574	0.001	-0.003	4.489	0.007	0.017

This table documents bias and variance of merger simulation outputs over 1,000 simulated datasets for the different numerical integration methods and numbers of integration nodes I_t considered in Table A2.

Table OA15: Merger Simulation: Problem Scaling

						True Median Value			Median Bias			Median Absolute Error		
Simulation	Supply	T	J_f	F_t	Seconds	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS	$\Delta\overline{\text{HHI}}$	ΔPS	ΔCS
Simple	No	20	$\{3,4,5\}$	$\{2,5,10\}$	0.8	1,114.102	0.051	-0.168	1.690	0.009	-0.036	8.134	0.012	0.044
Simple	No	100	$\{3,4,5\}$	$\{2,5,10\}$	3.8	1,062.686	0.253	-0.841	-0.365	0.007	-0.043	3.945	0.026	0.084
Simple	No	20	$\{15,20,25\}$	$\{2,5,10\}$	1.2	1,045.918	0.223	-0.488	0.939	0.010	-0.020	5.665	0.025	0.052
Simple	No	20	$\{3,4,5\}$	$\{10,25,50\}$	1.2	67.627	0.013	-0.024	0.005	0.001	-0.001	0.108	0.002	0.003
Simple	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	2.2	1,114.102	0.051	-0.168	0.252	-0.001	-0.002	8.493	0.008	0.027
Simple	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	8.9	1,062.686	0.253	-0.841	-0.787	-0.006	-0.000	3.803	0.019	0.060
Simple	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	9.6	1,045.918	0.223	-0.488	0.902	-0.000	0.001	5.604	0.021	0.046
Simple	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	9.5	67.627	0.013	-0.024	0.005	0.000	-0.000	0.102	0.001	0.003
Complex	No	20	$\{3,4,5\}$	$\{2,5,10\}$	1.6	1,106.325	0.060	-0.197	4.374	0.007	-0.033	15.943	0.016	0.057
Complex	No	100	$\{3,4,5\}$	$\{2,5,10\}$	7.0	1,051.334	0.300	-0.995	-0.908	0.012	-0.056	6.650	0.031	0.110
Complex	No	20	$\{15,20,25\}$	$\{2,5,10\}$	3.0	1,030.739	0.246	-0.541	2.540	0.008	-0.022	14.797	0.028	0.064
Complex	No	20	$\{3,4,5\}$	$\{10,25,50\}$	3.1	67.278	0.015	-0.027	-0.016	0.001	-0.002	0.321	0.002	0.004
Complex	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	4.7	1,106.325	0.060	-0.197	-1.640	0.001	-0.004	12.636	0.010	0.033
Complex	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	17.2	1,051.334	0.300	-0.995	-0.507	-0.004	0.000	6.018	0.022	0.072
Complex	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	26.4	1,030.739	0.246	-0.541	0.495	-0.001	-0.001	14.409	0.022	0.048
Complex	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	25.8	67.278	0.015	-0.027	-0.015	0.000	-0.000	0.296	0.001	0.003
RCNL	No	20	$\{3,4,5\}$	$\{2,5,10\}$	4.3	926.219	0.069	-0.157	0.213	0.014	-0.034	4.817	0.016	0.039
RCNL	No	100	$\{3,4,5\}$	$\{2,5,10\}$	18.2	905.388	0.349	-0.800	-1.843	0.016	-0.049	2.761	0.031	0.078
RCNL	No	20	$\{15,20,25\}$	$\{2,5,10\}$	6.0	916.229	0.158	-0.265	4.831	0.007	-0.015	4.870	0.017	0.028
RCNL	No	20	$\{3,4,5\}$	$\{10,25,50\}$	6.0	57.511	0.008	-0.012	0.078	0.000	-0.001	0.138	0.001	0.001
RCNL	Yes	20	$\{3,4,5\}$	$\{2,5,10\}$	9.3	926.219	0.069	-0.157	-1.574	0.001	-0.003	4.489	0.007	0.017
RCNL	Yes	100	$\{3,4,5\}$	$\{2,5,10\}$	35.3	905.388	0.349	-0.800	-2.452	0.000	-0.007	2.857	0.016	0.036
RCNL	Yes	20	$\{15,20,25\}$	$\{2,5,10\}$	36.5	916.229	0.158	-0.265	4.658	-0.003	0.002	4.665	0.014	0.023
RCNL	Yes	20	$\{3,4,5\}$	$\{10,25,50\}$	37.3	57.511	0.008	-0.012	0.071	-0.000	-0.000	0.129	0.001	0.001

This table documents bias and variance of merger simulation outputs over 1,000 simulated datasets for the different problem sizes considered in Table OA6.

OA8. Optimization

Table 6 documents that the vast majority of runs converge to a local optima, regardless of optimization routine. In Table OA16 we report analogous results for more routines. From Knitro, we add the Active Set (sequential linear-quadratic programming) and SQP (sequential quadratic programming) algorithms. From SciPy, we add an additional version of TNC (truncated Newton algorithm) configured with a gradient-based norm.

For completeness' sake, we replicate this table with sums of characteristics BLP instruments instead of feasible optimal instruments in Table OA17, and we document the impact of algorithm choice on bias and variance of parameter estimates in Table OA18. Results are very similar across algorithms.

Table OA16: Optimization Algorithms: Additional Routines

							Percent of Runs		Median, First GMM Step			
Simulation	Supply	$\lvert\theta_2\rvert$	Software	Algorithm	Gradient	Termination	Converged	PSD Hessian	Seconds	Evaluations	$q = \bar{g}'W\bar{g}$	$\lvert\lvert\nabla q\rvert\rvert_\infty$
Simple	No	1	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.2	4	1.10E-08	8.30E-07
Simple	No	1	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.2	4	8.85E-09	7.74E-07
Simple	No	1	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.3	4	1.10E-08	8.26E-07
Simple	No	1	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.2	4	8.19E-09	7.28E-07
Simple	No	1	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.6	11	1.58E-08	1.03E-06
Simple	No	1	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.3	4	3.88E-11	4.90E-08
Simple	No	1	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	99.9%	99.8%	0.6	10	3.89E-24	9.61E-15
Simple	No	1	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	66.5%	100.0%	19.6	115	1.08E-24	4.70E-15
Simple	Yes	2	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.6	5	2.17E-06	3.54E-06
Simple	Yes	2	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.6	4	2.17E-06	3.55E-06
Simple	Yes	2	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.5	5	2.17E-06	3.53E-06
Simple	Yes	2	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.4	4	2.18E-06	3.32E-06
Simple	Yes	2	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	2.0	11	2.61E-06	5.26E-06
Simple	Yes	2	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	0.9	8	2.06E-06	9.84E-07
Simple	Yes	2	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	99.8%	100.0%	2.2	18	2.15E-06	5.12E-11
Simple	Yes	2	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	53.3%	100.0%	31.7	251	2.14E-06	9.69E-13
Complex	No	3	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	96.9%	0.7	6	1.92E-07	4.11E-06
Complex	No	3	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	96.9%	0.6	6	1.90E-07	3.90E-06
Complex	No	3	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	96.9%	0.7	6	1.92E-07	4.12E-06
Complex	No	3	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	93.6%	0.4	6	1.83E-07	3.59E-06
Complex	No	3	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	96.5%	1.9	26	2.25E-07	5.14E-06
Complex	No	3	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	98.1%	0.6	6	2.41E-08	1.18E-06
Complex	No	3	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	100.0%	86.9%	1.8	20	5.08E-20	1.61E-12
Complex	No	3	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	54.0%	74.6%	30.1	275	1.18E-24	1.39E-14
Complex	Yes	4	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	93.7%	1.9	9	3.20E-06	6.11E-06
Complex	Yes	4	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	93.1%	1.9	9	3.18E-06	5.93E-06
Complex	Yes	4	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	93.9%	2.0	9	3.18E-06	6.11E-06
Complex	Yes	4	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	94.1%	1.4	9	3.12E-06	5.57E-06
Complex	Yes	4	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	94.2%	4.6	28	3.19E-06	6.36E-06
Complex	Yes	4	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	93.5%	2.0	11	2.75E-06	2.26E-06
Complex	Yes	4	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	99.5%	99.5%	5.7	31	2.87E-06	4.02E-10
Complex	Yes	4	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	45.5%	99.5%	64.6	480	2.80E-06	1.83E-12

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							Percent of Runs		Median, First GMM Step			
Simulation	Supply	$\lvert\theta_2\rvert$	Software	Algorithm	Gradient	Termination	Converged	PSD Hessian	Seconds	Evaluations	$q = \bar{g}'W\bar{g}$	$\lvert\lvert\nabla q\rvert\rvert_\infty$
RCNL	No	2	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	1.7	10	1.67E-08	5.72E-06
RCNL	No	2	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	1.8	11	2.84E-09	2.22E-06
RCNL	No	2	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	1.8	11	2.10E-09	1.96E-06
RCNL	No	2	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	99.9%	1.9	11	1.52E-09	1.47E-06
RCNL	No	2	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	4.3	25	2.64E-09	1.95E-06
RCNL	No	2	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	99.9%	1.7	10	8.27E-10	8.77E-07
RCNL	No	2	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	100.0%	99.6%	4.1	22	3.56E-19	1.16E-11
RCNL	No	2	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	56.4%	98.7%	60.5	243	1.27E-25	2.53E-14
RCNL	Yes	3	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	3.4	13	2.83E-06	9.95E-06
RCNL	Yes	3	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	3.7	14	2.78E-06	5.44E-06
RCNL	Yes	3	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	3.7	14	2.77E-06	4.43E-06
RCNL	Yes	3	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	3.2	12	2.66E-06	3.85E-06
RCNL	Yes	3	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	9.3	37	2.73E-06	4.41E-06
RCNL	Yes	3	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	100.0%	3.6	14	2.71E-06	1.96E-06
RCNL	Yes	3	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	100.0%	100.0%	7.2	24	2.95E-06	2.24E-09
RCNL	Yes	3	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	39.0%	100.0%	115.2	423	2.93E-06	4.60E-12

Like Table 6, this table also documents optimization convergence statistics over 1,000 simulated datasets, but for a larger number of optimization algorithms.

Table OA17: Optimization Algorithms: Sums of Characteristics BLP Instruments

							Percent of Runs		Median, First GMM Step			
Simulation	Supply	$\lvert\theta_2\rvert$	Software	Algorithm	Gradient	Termination	Converged	PSD Hessian	Seconds	Evaluations	$q = \bar{g}'W\bar{g}$	$\lvert\lvert\nabla q\rvert\rvert_\infty$
Simple	No	1	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	98.1%	0.1	2	4.28E-06	2.97E-06
Simple	No	1	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	97.4%	0.1	2	4.31E-06	3.34E-06
Simple	No	1	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	98.1%	0.1	2	4.31E-06	2.97E-06
Simple	No	1	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	97.1%	0.1	2	4.35E-06	3.36E-06
Simple	No	1	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	96.5%	0.2	2	4.95E-06	4.12E-06
Simple	No	1	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	98.6%	0.2	4	2.76E-06	6.70E-07
Simple	No	1	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	99.8%	99.7%	0.8	10	2.57E-06	4.01E-15
Simple	No	1	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	80.0%	99.9%	14.8	117	2.57E-06	1.39E-13
Simple	Yes	2	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	99.2%	0.4	4	8.46E-06	5.01E-06
Simple	Yes	2	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	99.0%	0.4	4	8.53E-06	4.97E-06
Simple	Yes	2	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	99.3%	0.4	4	8.44E-06	5.00E-06
Simple	Yes	2	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	98.3%	0.3	3	8.56E-06	5.45E-06
Simple	Yes	2	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	98.6%	1.0	9	9.28E-06	6.91E-06
Simple	Yes	2	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	99.4%	0.6	6	6.35E-06	1.54E-06
Simple	Yes	2	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	99.9%	99.9%	2.2	16	6.10E-06	1.78E-11
Simple	Yes	2	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	56.3%	99.9%	33.3	249	6.10E-06	4.54E-13
Complex	No	3	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	59.8%	0.4	4	5.31E-06	4.65E-06
Complex	No	3	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	59.4%	0.4	4	5.34E-06	4.69E-06
Complex	No	3	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	59.7%	0.4	4	5.31E-06	4.65E-06
Complex	No	3	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	58.7%	0.3	4	5.36E-06	4.14E-06
Complex	No	3	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	60.9%	0.8	9	5.51E-06	5.34E-06
Complex	No	3	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	67.4%	0.4	5	4.54E-06	1.84E-06
Complex	No	3	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	99.7%	99.6%	3.7	35	3.99E-06	7.60E-12
Complex	No	3	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	69.1%	99.8%	29.3	302	3.99E-06	5.95E-13
Complex	Yes	4	Knitro	Interior/Direct	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	76.4%	1.4	8	1.15E-05	6.75E-06
Complex	Yes	4	Knitro	Active Set	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	77.9%	1.4	7	1.16E-05	6.62E-06
Complex	Yes	4	Knitro	SQP	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	76.4%	1.4	8	1.15E-05	6.71E-06
Complex	Yes	4	SciPy	L-BFGS-B	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	74.9%	1.1	7	1.15E-05	6.23E-06
Complex	Yes	4	SciPy	BFGS	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	79.9%	2.7	11	1.17E-05	7.05E-06
Complex	Yes	4	SciPy	TNC	Yes	$\lvert\lvert\nabla q\rvert\rvert_\infty$	100.0%	82.8%	1.7	9	9.55E-06	2.90E-06
Complex	Yes	4	SciPy	TNC	Yes	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	100.0%	100.0%	6.8	37	9.25E-06	3.96E-10
Complex	Yes	4	SciPy	Nelder-Mead	No	$\lvert\lvert\theta_2^n - \theta_2^{n-1}\rvert\rvert_\infty$	44.9%	100.0%	62.5	455	9.25E-06	2.15E-12

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							Percent of Runs		Median, First GMM Step			
Simulation	Supply	$\|\theta_2\|$	Software	Algorithm	Gradient	Termination	Converged	PSD Hessian	Seconds	Evaluations	$q = \bar{g}'W\bar{g}$	$\|\|\nabla q\|\|_\infty$
RCNL	No	2	Knitro	Interior/Direct	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	91.4%	0.7	4	4.99E-06	8.35E-06
RCNL	No	2	Knitro	Active Set	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	91.2%	0.8	5	4.98E-06	4.09E-06
RCNL	No	2	Knitro	SQP	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	92.1%	0.8	5	4.98E-06	3.78E-06
RCNL	No	2	SciPy	L-BFGS-B	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.9%	0.8	5	4.93E-06	3.97E-06
RCNL	No	2	SciPy	BFGS	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.6%	1.8	9	5.13E-06	3.67E-06
RCNL	No	2	SciPy	TNC	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	93.9%	1.2	6	3.60E-06	2.39E-06
RCNL	No	2	SciPy	TNC	Yes	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	99.9%	99.8%	4.5	23	3.32E-06	9.84E-11
RCNL	No	2	SciPy	Nelder-Mead	No	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	60.9%	99.9%	57.7	251	3.28E-06	1.76E-12
RCNL	Yes	3	Knitro	Interior/Direct	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.3%	1.6	6	1.12E-05	9.54E-06
RCNL	Yes	3	Knitro	Active Set	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.3%	1.8	6	1.10E-05	6.29E-06
RCNL	Yes	3	Knitro	SQP	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.6%	1.8	6	1.10E-05	5.73E-06
RCNL	Yes	3	SciPy	L-BFGS-B	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.6%	1.7	6	1.10E-05	5.40E-06
RCNL	Yes	3	SciPy	BFGS	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	90.1%	5.2	22	1.07E-05	5.69E-06
RCNL	Yes	3	SciPy	TNC	Yes	$\|\|\nabla q\|\|_\infty$	100.0%	95.8%	2.4	8	9.35E-06	3.04E-06
RCNL	Yes	3	SciPy	TNC	Yes	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	100.0%	100.0%	7.4	25	8.94E-06	5.13E-10
RCNL	Yes	3	SciPy	Nelder-Mead	No	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	39.9%	100.0%	109.2	418	8.94E-06	2.38E-12

Like Tables 6 and OA16, this table also documents optimization convergence statistics over 1,000 simulated datasets for different optimization algorithms. Instead of feasible optimal instruments, the problems solved for this table use only sums of characteristics BLP instruments.

Table OA18: Optimization Algorithms: Parameter Estimates

						True Value				Median Bias				Median Absolute Error			
Simulation	Supply	Software	Algorithm	Termination	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	No	Knitro	Interior/Direct	$\|\|\nabla q\|\|_\infty$	0.9	-1	3			0.190	-0.039			0.244	0.169		
Simple	No	Knitro	Active Set	$\|\|\nabla q\|\|_\infty$	0.9	-1	3			0.195	-0.045			0.246	0.170		
Simple	No	Knitro	SQP	$\|\|\nabla q\|\|_\infty$	0.9	-1	3			0.189	-0.038			0.244	0.168		
Simple	No	SciPy	L-BFGS-B	$\|\|\nabla q\|\|_\infty$	0.8	-1	3			0.191	-0.044			0.245	0.187		
Simple	No	SciPy	BFGS	$\|\|\nabla q\|\|_\infty$	1.3	-1	3			0.191	-0.051			0.249	0.180		
Simple	No	SciPy	TNC	$\|\|\nabla q\|\|_\infty$	1.0	-1	3			0.195	-0.045			0.246	0.168		
Simple	No	SciPy	TNC	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	1.8	-1	3			0.191	-0.045			0.245	0.172		
Simple	No	SciPy	Nelder-Mead	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	18.3	-1	3			0.193	-0.046			0.245	0.171		
Simple	Yes	Knitro	Interior/Direct	$\|\|\nabla q\|\|_\infty$	2.4	-1	3			0.029	0.000			0.168	0.182		
Simple	Yes	Knitro	Active Set	$\|\|\nabla q\|\|_\infty$	2.4	-1	3			0.034	-0.001			0.169	0.173		
Simple	Yes	Knitro	SQP	$\|\|\nabla q\|\|_\infty$	2.4	-1	3			0.029	-0.000			0.167	0.182		
Simple	Yes	SciPy	L-BFGS-B	$\|\|\nabla q\|\|_\infty$	2.3	-1	3			0.022	-0.012			0.181	0.185		
Simple	Yes	SciPy	BFGS	$\|\|\nabla q\|\|_\infty$	4.1	-1	3			0.047	-0.030			0.187	0.206		
Simple	Yes	SciPy	TNC	$\|\|\nabla q\|\|_\infty$	3.0	-1	3			0.027	-0.011			0.172	0.172		
Simple	Yes	SciPy	TNC	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	5.9	-1	3			0.013	-0.015			0.169	0.175		
Simple	Yes	SciPy	Nelder-Mead	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	50.8	-1	3			0.012	-0.015			0.169	0.175		
Complex	No	Knitro	Interior/Direct	$\|\|\nabla q\|\|_\infty$	1.9	-1	3	0.2		0.153	-0.106	-0.006		0.241	0.199	0.122	
Complex	No	Knitro	Active Set	$\|\|\nabla q\|\|_\infty$	1.8	-1	3	0.2		0.164	-0.114	-0.009		0.235	0.195	0.124	
Complex	No	Knitro	SQP	$\|\|\nabla q\|\|_\infty$	1.9	-1	3	0.2		0.153	-0.106	-0.004		0.241	0.199	0.122	
Complex	No	SciPy	L-BFGS-B	$\|\|\nabla q\|\|_\infty$	1.7	-1	3	0.2		0.168	-0.112	-0.035		0.243	0.197	0.159	
Complex	No	SciPy	BFGS	$\|\|\nabla q\|\|_\infty$	3.8	-1	3	0.2		0.133	-0.116	-0.016		0.218	0.207	0.136	
Complex	No	SciPy	TNC	$\|\|\nabla q\|\|_\infty$	2.0	-1	3	0.2		0.158	-0.101	0.004		0.240	0.183	0.112	
Complex	No	SciPy	TNC	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	6.0	-1	3	0.2		0.150	-0.110	0.017		0.238	0.199	0.125	
Complex	No	SciPy	Nelder-Mead	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	51.0	-1	3	0.2		0.154	-0.099	0.022		0.230	0.204	0.078	
Complex	Yes	Knitro	Interior/Direct	$\|\|\nabla q\|\|_\infty$	5.4	-1	3	0.2		-0.023	-0.070	-0.007		0.178	0.192	0.125	
Complex	Yes	Knitro	Active Set	$\|\|\nabla q\|\|_\infty$	5.4	-1	3	0.2		-0.019	-0.061	-0.018		0.180	0.190	0.123	
Complex	Yes	Knitro	SQP	$\|\|\nabla q\|\|_\infty$	5.5	-1	3	0.2		-0.021	-0.067	-0.006		0.181	0.191	0.126	
Complex	Yes	SciPy	L-BFGS-B	$\|\|\nabla q\|\|_\infty$	5.2	-1	3	0.2		-0.030	-0.052	-0.047		0.185	0.188	0.147	
Complex	Yes	SciPy	BFGS	$\|\|\nabla q\|\|_\infty$	9.7	-1	3	0.2		-0.046	-0.072	-0.045		0.190	0.197	0.144	
Complex	Yes	SciPy	TNC	$\|\|\nabla q\|\|_\infty$	6.2	-1	3	0.2		-0.006	-0.045	-0.015		0.172	0.178	0.116	
Complex	Yes	SciPy	TNC	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	14.7	-1	3	0.2		-0.032	-0.058	0.013		0.182	0.189	0.146	
Complex	Yes	SciPy	Nelder-Mead	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	125.5	-1	3	0.2		-0.032	-0.063	0.015		0.183	0.189	0.140	

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						True Value				Median Bias				Median Absolute Error			
Simulation	Supply	Software	Algorithm	Termination	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
RCNL	No	Knitro	Interior/Direct	$\|\|\nabla q\|\|_\infty$	4.5	-1	3		0.5	0.179	-0.047		-0.005	0.217	0.161		0.022
RCNL	No	Knitro	Active Set	$\|\|\nabla q\|\|_\infty$	4.6	-1	3		0.5	0.179	-0.024		-0.007	0.217	0.161		0.022
RCNL	No	Knitro	SQP	$\|\|\nabla q\|\|_\infty$	4.7	-1	3		0.5	0.178	-0.029		-0.007	0.216	0.160		0.022
RCNL	No	SciPy	L-BFGS-B	$\|\|\nabla q\|\|_\infty$	4.7	-1	3		0.5	0.178	-0.027		-0.007	0.218	0.157		0.023
RCNL	No	SciPy	BFGS	$\|\|\nabla q\|\|_\infty$	8.1	-1	3		0.5	0.174	-0.015		-0.008	0.217	0.159		0.022
RCNL	No	SciPy	TNC	$\|\|\nabla q\|\|_\infty$	4.8	-1	3		0.5	0.176	-0.019		-0.007	0.215	0.159		0.021
RCNL	No	SciPy	TNC	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	10.3	-1	3		0.5	0.177	-0.010		-0.008	0.214	0.166		0.021
RCNL	No	SciPy	Nelder-Mead	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	94.0	-1	3		0.5	0.177	-0.008		-0.008	0.215	0.167		0.021
RCNL	Yes	Knitro	Interior/Direct	$\|\|\nabla q\|\|_\infty$	9.4	-1	3		0.5	0.016	-0.029		0.003	0.112	0.174		0.021
RCNL	Yes	Knitro	Active Set	$\|\|\nabla q\|\|_\infty$	9.9	-1	3		0.5	0.011	-0.025		0.002	0.110	0.173		0.020
RCNL	Yes	Knitro	SQP	$\|\|\nabla q\|\|_\infty$	9.7	-1	3		0.5	0.012	-0.023		0.002	0.110	0.169		0.020
RCNL	Yes	SciPy	L-BFGS-B	$\|\|\nabla q\|\|_\infty$	9.3	-1	3		0.5	0.009	-0.032		0.003	0.109	0.163		0.019
RCNL	Yes	SciPy	BFGS	$\|\|\nabla q\|\|_\infty$	18.6	-1	3		0.5	0.012	-0.021		0.002	0.113	0.173		0.020
RCNL	Yes	SciPy	TNC	$\|\|\nabla q\|\|_\infty$	10.3	-1	3		0.5	0.007	-0.002		0.001	0.108	0.156		0.020
RCNL	Yes	SciPy	TNC	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	18.5	-1	3		0.5	0.004	0.005		0.001	0.108	0.169		0.021
RCNL	Yes	SciPy	Nelder-Mead	$\|\|\theta_2^n - \theta_2^{n-1}\|\|_\infty$	223.5	-1	3		0.5	0.003	0.005		0.001	0.108	0.170		0.021

This table documents bias and variance of parameter estimates over 1,000 simulated datasets for different optimization algorithms. It is created from the same results as Table OA16 after a second GMM step.

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